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Summary of the book „How to Construct High IQ Math ...” by Krzysztof Zawisza

The subject of Krzysztof Zawisza's book "How to construct high IQ math, create new superior physics and gain control over space and time in the next 20 years" is **a program to base mathematical and natural sciences directly on the basic principles of logic, i.e. on the principles of correct thinking**. Such an approach to scientific methodology characterized the creators of science and human thought as such (Pythagoreans, Plato, Aristotle) and the creators of modern science and thought (Leibniz, Hegel and others). However, the fully rational, reasonable approach requires the ability to think in a purely abstract, logical-conceptual way, which (as the author shows) only people with an IQ of 175+ SD15 (like Pythagoras, Plato, Gottfried Leibniz or Niels Bohr) are fully capable of. Therefore, along with the increase in the democratization of science and, consequently, the decrease in the average intelligence of scientists, this approach was abandoned in favour of an intellectually less demanding empirical and quasi-empirical approach.

However, empirical and quasi-empirical research methods are cognitive methods of common sense, not reason. Common sense is the highest expression of the social (i.e. herd) instinct and is based on popular belief and collective thinking. The common sense's method is association. Meanwhile, the method of reason is conceptual thinking, which consists in abstracting from associations. Therefore common sense is essentially opposed to reason. For it is the shape, that is, the end or limit of reason, and reason ends with common sense. In the process of self-egalitarianization², which has been going on since ancient times³, science has rejected reason and ceased to strive for a deep understanding of reality. The constant decrease in the average intelligence of scientists and the consequent tendency of today's scholars to inhibit *reason* is an obvious *reason* for the inhibition of scientific progress⁴, which we have been observing over the last few decades, especially in theoretical physics⁵.

The first creators of both ancient and modern science followed the trail of reasonable necessity of the objects studied by science and rational thinking. As Diogenes Laertius writes about the views of the Pythagorean Philolaus (V century BC):

Δοκεῖ δ' αὐτῷ πάντα ἀνάγκη καὶ ἁρμονία γίνεσθαι. καὶ τὴν γῆν κινεῖσθαι κατὰ κύκλον πρῶτον εἰπεῖν.⁶

[„His doctrine is that all things are brought about by necessity and in harmonious inter-relation”⁷].

About 2 thousand years after Philolaus, Galileo Galilei [estimated IQ 180+] wrote almost the same:

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² Gastfriend, E. *90% of All the Scientists That Ever Lived Are Alive Today*, Future of Life Institute, <https://futureoflife.org/2015/11/05/90-of-all-the-scientists-that-ever-lived-are-alive-today/?cn-reloaded=1>.

³ According to the author's estimates, the lower limit of intelligence for members of the Pythagorean Association was probably five standard deviations above today's average. These five standard deviations from the average intelligence are today the upper limit for scientists, beyond which the possessor of such high intelligence becomes incomprehensible to the average members of the scientific community, as a result of which such people today never make a career in science.

⁴ Carlson, N. “Other Than in Computers, Civilization Basically Stopped Progressing in the 1960s”, *Business Insider*, Jun 10, 2012, 3:22 PM, <https://www.businessinsider.com/other-than-in-computers-civilization-basically-stopped-progressing-in-the-1960s-2012-6?IR=T>.

⁵ Hossenfelder, S. “The Present Phase of Stagnation in the Foundations of Physics Is Not Normal”, *Nautilus*, November 23, 2018, https://getpocket.com/explore/item/the-present-phase-of-stagnation-in-the-foundations-of-physics-is-not-normal?utm_source=pocket-newtab&fbclid=IwAR1NA6Yke3TMekebRqsbyMSBwOPjoHE-3DFwAOko2azmlFvRPMvD6EPnQYw .

⁶ Diog. Laert. VIII, 85.

⁷ Diogenes Laertius, *Lives of Eminent Philosophers* with an English Translation by D. Hicks, M.A. London: William Heinemann, New York: G.P. Putnam's Sons, MCMXXV. Cf. Boeckh, A. *Philolaos des Pythagoreers Lehren nebst den Bruchstücken seines Werkes*, Berlin, 1819.

“[Without recourse to experience: *sanza sperienza*] I am assured that the effect will ensue as I tell you; for so it is necessary that it should [...]”⁸.

Yet before Galileo, Leonardo da Vinci [estimated IQ 180+] stated:

„In nature there is no effect without cause; once the cause is understood there is no need to test it by experience”^{9,10}.

In a similar (strictly deductive) spirit spoke out Johannes Kepler [estimated IQ 175]:

„For I have no hesitation in asserting that everything, that Copernicus has demonstrated *a posteriori* on the basis of observations interpreted geometrically, may be demonstrated *a priori* without any subtlety of logic”¹¹.

However, even Copernicus [est. IQ 180 SD15] based in fact his research on logically subtle *a priori* argumentation:

“**The universe is spherical.** First of all, we must note that the universe is spherical. The reason is either that, of all forms, the sphere is the most perfect, needing no joint and being a complete whole, which can be neither increased nor diminished; or that it is the most capacious of figures, best suited to enclose and retain all things; or even that all the separate parts of the universe, I mean the sun, moon, planets and stars, are seen to be of this shape; or **that wholes strive to be circumscribed by this boundary, as is apparent in drops of water and other fluid bodies when they seek to be self-contained.** Hence no one will question the attribution of this form to the divine bodies”¹².

Having rejected human *reason*, modern science has ceased to understand the *reason* of man and the world. To justify this state of affairs, theorists of science irrationally hold that reality is rationally unknowable (that is, that reality would have to be essentially irrational). According to the same theoreticians of science, the human mind is allegedly extremely limited and supposedly does not understand anything, and some scientific problems are supposedly so problematic that they allegedly cannot be solved and can only be circumvented. We usually bypass the truth when we feel fear of the truth and cannot bear the brilliance of naked truth.

The allegedly unsolvable scientific problems (which supposedly can only be "circumvented" or "bypassing") would include such issues as the problem of antinomy in logic (especially the problem of the antinomy of the Liar), the problem of division by zero in mathematics, or the construction of the Unified Field Theory in physics. However, as the Author shows, these problems - which turned out to be incomprehensible from the common-sense point of view - have their base and reason in abstract thinking and logic and are clearly solvable by reason.

⁸ Galilei, G. *Dialogues on two world systems*, tr. Salusbury. T., http://archimedes.mpiwg-berlin.mpg.de/cgi-bin/toc/toc.cgi?step=thumb&dir=galil_syste_065_en_1661. In Stillman Drake's translation it reads: "Without experiment, I am sure that the effect will happen as I tell you, because it must happen that way" [Suppe, F. [ed.], *The Structure of Scientific Theories*, Univ. of Illinois Press, 1977].

⁹ Da Vinci, L. *The Notebooks of Leonardo Da Vinci*, <http://www.gutenberg.org/etext/5000>. Original Italian text: "Nessun effetto è in natura senza ragione; intendi la ragione e non ti bisogna sperienza". MacCurdy's translation: "There is no result in nature without a cause; understand the cause and you will have no need of the experiment" ["Philosophy", in *The Notebooks of Leonardo da Vinci*, trans. E. MacCurdy, (1938) Vol. 1, 70.]. My own literal translation: "No effect is in nature without **reason**; you understand the **reason** and you don't need experience". Then Leonardo is actually saying something like this: "Once you get inside the reason of human reason you will not have any reason for empirical experience".

¹⁰ Cf. Luporini, C. *La mente di Leonardo*, G.C. Sansoni Editore, Firenze, 1953.

¹¹ Kepler, J. *Mysterium cosmographicum*, Tubingen, 1596. The full sentence in Latin: "Neque dubito affirmare, quicquid a posteriori Copernicus collegit, et visu demonstravit, mediantibus Geometricis axiomatis, id omne vel ipso Aristotele teste, si viveret (quod frequenter optat Rheticus) a priori demonstrari posse nullis ambagibus".

¹² The Edward Rosen translation of *De Revolutionibus Orbium Coelestium*, Libri VI, Polish Scientific Publications, Warsaw, 1978.

1. The Liar Paradox

The antinomy of the Liar was considered by Aristotle - as he wrote about it in "On Sophistical Refutations" - to be only a rather trivial sophism. Nevertheless, the logicians of subsequent epochs moved further and further away from the view that there was a simple solution to this paradox, and consequently focused on finding a way around this apparent antinomy, instead of on its elimination. Meanwhile, from a rational point of view, the explanation of this paradox is simple. The author begins with an analysis of the non-empirical form of the Liar sentence given by Alfred Tarski:

„Let S be any sentence beginning with the words "Every sentence." We correlate with S a new sentence S* by subjecting S to the following two modifications: we replace in S the first word, "Every," by "The"; and we insert after the second word, "sentence," the whole sentence S enclosed in quotation marks. Let us agree to call the sentence S "(self-)applicable" or "non-(self-)applicable" dependent on whether the correlated sentence S* is true or false. Now consider the following sentence:

Every sentence is non-applicable.

It can easily be shown that the sentence just stated must be both applicable and nonapplicable; hence a contradiction. It may not be quite clear in what sense this formulation of the antinomy does not involve an empirical premiss; however, I shall not elaborate on this point”¹³.

Then the Author contradicts the sentence quoted by A. Tarski. The negation of the sentence "Every sentence is non-applicable" will be the sentence "There is an applicable sentence", which turns out to be true. For let's take S: = "Every sentence is the same sentence as it is". Then S* = "The sentence 'every sentence is the same sentence as it is' is the same sentence as it is". The sentence S* is true by virtue of the principle of identity; therefore S is an applicable sentence. So (by giving an example) we come to the statement: there is an applicable sentence.

Therefore, since the negation of the Liar's Sentence (at least in its non-empirical form) is a meaningful and true sentence, the same Liar's Sentence should also be a meaningful but false sentence.

Then the Author, using the scientific methodology of such creators of science as Aristotle or Leibniz, makes a purely rational, logical and conceptual analysis of the Liar's Sentence.

- a) Telling a lie is telling something other than the truth.
- b) Truth is reality (every truth is true, i.e. real, and every reality - in order to be a reality - must really be true and truly real; therefore, the terms "truth" and "reality" are the same as to their content).
- c) The reality is what is.
- d) So if the Liar says that what he says is a lie, he is saying that what he says is different from what is.
- e) So the Liar says that he says something other than is contained in what he says.
- f) So the Liar is saying that he is saying what is not in his sentence, i.e. that he is saying something different from what he is saying.
- g) And if he says something other than what he says, he is saying that he is not saying¹⁴ (because you can only say what you say).
- h) And if the Liar says he doesn't say, he's obviously Lying (because he contradicts himself and denies the facts).
- i) So the Liar is lying, and the famous **Liar Sentence is simply a false sentence** (in the simplest, i.e. true sense of falsehood).

¹³ Tarski, A. "The Semantic Conception of Truth: and the Foundations of Semantics", *Philosophy and Phenomenological Research*, Vol. 4, No. 3 (Mar., 1944), pp. 341-376.

¹⁴ Conclusion g) comes from Jakub Nowak (TETRIQ High IQ Society), who, while reviewing Krzysztof Zawisza's work, arrived at such a simple formulation.

The Author shows that the above conclusion can also be reached from any other logical perspective. Also formalizing the Liar Sentence in the correct way, we get the conclusion that this sentence is simply a meaningful and false sentence. The Author notes that the usual way of formalizing the Liar Sentence (which leads to the paradox)

$$z \leftrightarrow \neg z \quad (1a)$$

is incorrect, i.e. false. Because the Liar Sentence implies its own negation (the negation of the Liar Sentence is contained in the substance of the Liar Sentence), but the negation of the Liar Sentence does not imply the Liar Sentence (the negation of the Liar Sentence is the so-called Truth-teller Sentence: 'what I am now saying is true', in which the Liar's Sentence is not contained and not implied by). Thus, the truly formalization of the Liar's sentence entails only the following implication:

$$z \mapsto \neg z. \quad (1b)$$

Of course, as follows from the laws of propositional calculus, **the above implication is true if and only if the Liar's statement (z) is false.**

The author also proved that if we want to write the information about the Liar sentence correctly in the form of logical equivalence, we must do it as follows

$$z \leftrightarrow (\neg z \& z). \quad (1c)$$

This is because the Liar Sentence is not equivalent to its own negation, but it is equivalent to its own contradiction (the content of the Liar Sentence is the contradiction of that sentence to itself). (1c) also leads to the conclusion that **the Liar's Sentence is simply a false sentence.**

The Author notes that treating the Liar's Sentence as a false sentence in the classic sense of falsehood was postulated already at the end of the last century by the Polish logician Piotr Łukowski, a professor at the University of Lodz. However, Łukowski cannot prove his theorem for the general case, but only for the narrowed case of a certain formal representation of the Liar's Sentence¹⁵.

The Author also demonstrates that we've had the solution to the Liar paradox right under our noses for the last almost 2,500 years. It is contained in what and how we think about the Liar Sentence whenever we start to analyze it. Namely, as soon as we start thinking about the Liar Sentence, we fall seemingly into the vicious circle of thinking [*circulus vitiosus*]¹⁶: if we assume that the Liar's statement is false, it follows that it is true, and if we assume that it is true, it looks false. However, after a while of thinking (i.e. after a moment of logical reflection) we must now come to the conclusion that it doesn't lead to any "antinomy". If we consider the Liar's Sentence false only when we assumed it to be true, and as true - when we assumed it was false, it means that we considered the Liar's sentence with its falsehood as true, and we considered as false - the Liar's Sentence with the assumed truth about it. In this case, we *de facto* consider the falsehood assumed about the Liar's Sentence to be true, and the truth assumed about it as false. So, **although this simple fact was known by over 2,000 years, no logician or philosopher has been able to see clearly that Liar's Sentence behaves like any false sentence.** For every false sentence we can say that adjudication the truth about it is false, and that the falsehood adjudicated about it is true. For the logicians who analyze the Liar's Sentence, the same "naked" sentence is "merged into one" with its veracity or falsehood pronounced about it. In this way the logicians make a textbook *equivocation fallacy*.

We will now formalize the above observations using logical symbolism. Let z denote "Liar's Sentence", x - any truthful sentence, v - truth (more precisely - predicate: "is true"), f - false (predicate: "is false"), T - truth function, i.e. a function assigning to true propositions - the value of truth (here: 1), and to false propositions - the value of falsehood (here: 0) in two-valued logic. So we have (according to the redundant understanding of truth):

¹⁵ Łukowski, P. *Paradoxes*, Springer Science & Business Media, 2 June 2011.

¹⁶ Fenstad, J.E., Frolov, I.T., Hilpinen, R. *Logic, Methodology and Philosophy of Science VIII*, North-Holland, 1989.

$T(„v(x)”) = T(x)$ [we will omit the quotation marks in the brackets hereafter for simplicity].

In line with everything we said above, we immediately conclude:

$$(T(v(z)) = 0) \& (T(f(z)) = 1). \quad (2a)$$

Hence:

$$T(z) = 0. \quad (2b)$$

Conclusion: The sentence z (i.e. the Liar's Sentence) is a false sentence in the classical sense of falsehood.

In a similarly simple, logical-conceptual way, the Author solves the problem of the Circle of Liars ("1. The next sentence is true, 2. The previous sentence is false"). The Author shows that both sentences of the Circle of Liars are false, although they are false for different reasons.

In a similar way, i.e. by directly referring to the principles of logic, the author solves the problem of other known antinomies - both semantic and set-theoretic. These include the Richard antinomy or the Russell antinomy.

2. Division by zero

The problem of division by zero in mathematics is over 1,300 years old - since zero was discovered as a number in India. The first proposals for solutions went in the right direction (Brahmagupta in the 7th century AD wrote that zero divided by zero equals zero). Later, however, the deepening intellectual collapse in science and the incapacity of scientists for purely abstract - i.e. logical-conceptual - thinking, led to the fact that (as in the case of logical antinomies) the focus was not on solving this problem, but on attempts to circumvent it (the easiest, although logically unjustified, method of dealing with this problem is the so far binding in mathematics "prohibition of division by zero"¹⁷).

However, a simple logical analysis of the problem leads to the immediate conclusion that, from a purely rational point of view, division by zero equals zero. This results directly from the principle of non-contradiction, or more precisely from the ontological form of this principle, which is called the *principle of non-contradiction of being*.

For if we group r elements into zero groups (*partitive division*), then each of these nonexistent groups will contain exactly zero elements. For that which does not exist cannot contain anything¹⁸. Similarly, if we group any r elements into groups so that each of these groups contains zero elements (*measurement division*), we get zero groups as a result. For groups that contain no elements are not groups at all, and therefore do not exist at all. The number of non-existent groups is, of course, zero, because if something does not exist, then it does not exist at all - and therefore there is zero of it.

In other words, if we divide 12 chocolates fairly among 4 people (partitive division, sharing), each person will receive 3 chocolates. If we distribute 12 chocolates among some people so that each of them receives 4 chocolates (measurement division, housing), then there must be 3 people. So $12/4 = 3$. If we distribute 12 chocolates evenly among 0 people, then none of the people will receive any chocolates (because there is no one who could receive these chocolates). Therefore, the number of chocolates obtained in this division is equal to zero. Similarly, if we distribute 12 chocolates fairly among some people in such a way that each person gets zero chocolates, then the number of people in this distribution of chocolates must be equal to zero. Because if the number of these people was different from zero, then each of them would get some chocolates (or some part of the chocolate). So $12/0 = 0$.

¹⁷ Maor, E. "Thou Shalt Not Divide by Zero!", *Math Horizons*, 11:2 (2003), 16-19.

¹⁸ As the Roman poet and philosopher Titus Lucretius Carus logically necessarily noted in the first century B.C.E.: "Ex nihilo nihil fit", i.e. "Nothing can be made from nothing" [**nothing follows from nothing, or there is nothing followed by nothing**]; Lukretius, *De rerum natura*, 1, 149, 205; 2, 287.

Similarly, if we distribute zero chocolates equally among zero people, then the number of chocolates received by one person will also be equal to zero. First, there is no one person. Second, there are no chocolates to receive. So $0/0 = 0$.

Although it is fully true that in modern mathematics there is the so-called *vacuous truth theory*, which states that non-existent objects may allegedly possess existing things or features. In that concept, non-existent people could (supposedly) have some chocolates.

In accordance with the adopted axioms of ZF, for each sentence function $F(x)$ there is:

$$\forall x \in \emptyset: (F(x) \& \neg F(x)). \quad (3a)$$

Marking the subquantifier conjunction by $K(x) = F(x) \& \neg F(x)$ and applying to the sentence form $K(x)$ **the simplification rule** usually concludes from (3a) that for each sentence formula $F(x)$ it is true that

$$\forall x \in \emptyset: (F(x)). \quad (3b)$$

Sentence (3b) is usually formulated as "thesis about nonsensical (or mindless) truth": *vacuous truth statement*¹⁹. It says that "all elements of an empty set have every (any) property". A textbook example of such a "pointlessly true" statement is the saying that "all cell phones in the mouse hole are currently turned on". However, the mistake that underlies the concept of *vacuous truth* is, of course, that we take as a thesis (i.e. as truth) in sentence (3a) the sentence form $K(x) = F(x) \& \neg F(x)$, which for each value of x is a logically false sentence. This results directly from the logical principle of non-contradiction, which states that no statement (no sentence) can be truly both true and false²⁰. Therefore, if T denotes the truth function (assigning the number 1 to true sentences and the value 0 to false sentences), we can write

$$\forall x: T("K(x)") = 0. \quad (3c)$$

It should be emphasized that the propositional form (3a) as a whole is true, but the sentence form $K(x)$ is false for every x . Because non-existent objects ($x \in \emptyset$) are in other words false objects and therefore they are truly described only by false sentences.

Anyway, the sentence form $K(x)$ is a false sentence for every x and **we cannot apply the simplification rule to a false sentence** [according to the rules of deduction neither sentence p nor sentence q can be derived from the sentence $\neg(p \& q)$]. Thus the propositional form (3b) is burdened with the logical fallacy of *petitio principii* ("I demand a basis"). It fully shows that the concept of *vacuous truth* itself is truly vacuous i.e. absent-minded and impassive.

Therefore, if I don't have an account at *Deutsche Bank*, then telling anyone that I have a million euros in my account at *Deutsche Bank*, although in line with the concept of *vacuous truth*, in the light of the rules of right thinking would simply be wrong (and by any court it will be considered a fraud).

In order to implement the logically necessary result of division by zero into mathematics as a formal science, the formal algebraic definition of division must be generalized. In today's mathematical formalism, formally based on ZF²¹ axiomatics, we define the division of numbers as follows

$$\frac{a}{b} = a \cdot b^{-1}, b \neq 0. \quad (4)$$

Since *we define division as multiplication by an inverse element*, the reason for the alleged lack of logical admissibility of defining division by 0, as – in classical sense – there is no number inverse to 0, i.e. there is no number that multiplied by 0 will result in the neutral element of multiplication (i.e. *one*).

¹⁹Epp, S.S. *Discrete Mathematics with Applications. An Introduction to Mathematical Reasoning* [various editions].

²⁰ <https://web.stanford.edu/~bobonich/glances%20ahead/IV.excluded.middle.html>.

²¹ Fraenkel, A., Bar-Hillel, Y., Lévy, A. *Foundations of Set Theory*, North-Holland, 1973.

We should start from **generalization of the inverse element definition**. Classically, we define a neutral element of an algebraic operation as a property of numerical sets with a given operation²². Let S be a set with a specified binary operation \diamond .

Def. 1

For any $x, y \in S$ elements, the x element is called the inverse of y if and only if the following condition is met:

$$x \diamond y = y \diamond x = e,$$

where e is the neutral element of a given operation \diamond .

Now we will introduce the definition of a *generalized multiplication inverse element*.

Def. 2

Element x is called *the generalized inverse element* of y for the \diamond operation if the following condition is met:

$$(x \diamond y = y \diamond x = e) \vee (x = y = 0),$$

where 0 – neutral element of addition.

Replacing *def. 1* by *def. 2* in (4) **we immediately get numerical algebra with division by zero**.

However, the existence of the “prohibition of division by zero” is often justified as follows. Namely, because the value of the real function $f(x) = x^{-1}$ for $x \rightarrow 0$ tends to plus infinity for positive x -s, and to minus infinity for negative x -s, so there is no classically understood limit of this functions in zero [see figure below].

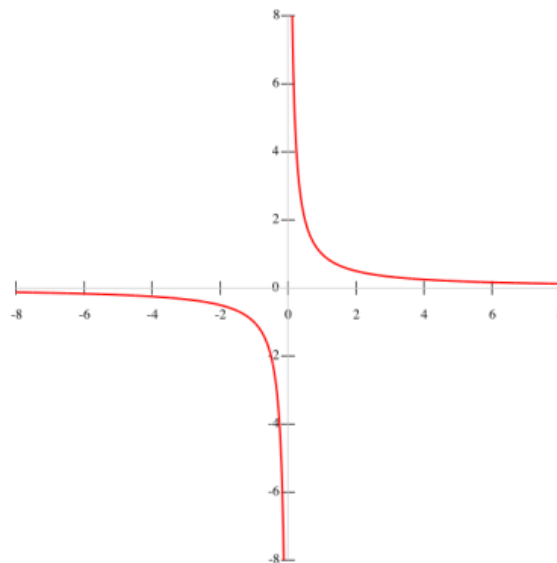


Figure 1 | Graph of the real function $f(x) = 1/x$ with an arbitrarily limited understanding of the concepts of the number zero and the dividing by zero [this restriction is a textbook logical error: a *reductive fallacy error*].

It should be noted, however, that the mean value of the function $f(x)$ (calculated from the Integral Mean Value Theorem) for both limited ranges $[-y, y] \setminus [-x, x]$ ($x < y; x, y \neq 0$), as well as for the whole domain $R \setminus 0$ (when we converge with x to zero, with y to infinity) is constant and equals 0. Although there is an indelible discontinuity of the function $f(x)$ in zero by a classic definition of a function's continuity, however, we can complete the domain of that function due to its average value, assuming $f(0) = 0$.

²² Howie, J. M. *Fundamentals of Semigroup Theory*, Clarendon Press, Oxford, 1995.

Also for that reason, **a zero equal result of division by zero in mathematics is a logically necessary result.**

The author also emphasizes that banning the result of dividing a complex number by zero (i.e. stating that the result of dividing a complex number by zero is not an element of the set of complex numbers) not only has no logical basis (*petitio principii*), but immediately leads to a contradiction. For if the result of dividing a complex number by zero is no complex number, then as a complex number it is nothing, i.e. it is zero (it is no thing and not a thing)²³. Hence, the following implication is logically necessary

$$\left(\frac{1}{0} \notin \mathcal{C}\right) \rightarrow \left(\frac{1}{0} = 0 \in \mathcal{C}\right). \quad (5)$$

The Author notes that the zero result of division by zero was postulated by the Japanese mathematician Saborou Saitoh, professor emeritus from Gunma University. However, Saitoh is unable to prove this result logically or to derive important applications from it²⁴.

In any case, knowing and understanding the fact that the result of division by zero is zero ends the supposed existence of many apparent paradoxes in both mathematics and physics. It immediately follows from this that the density of matter concentrated at a point, i.e. in a zero volume, is not "infinite" (as it is still believed today), but zero. Density is an amount of mass divided by some volume. So if the volume is not some but none (there is no volume, volume is zero), then we cannot talk about density. There is no density then, i.e. the density is also zero (i.e. none). In this way, the so-called singularities in physics, and physics itself finally becomes logical and coherent, i.e. scientific.

Some simple examples of positive consequences of allowing division by zero in physics

It will be shown below in the form of **Q&A** the solutions to the simplest physical problems by assuming that $0^{-1} = 0$.

1⁰ **Q.** What is the mass of a physical body that does not accelerate despite the action of a non-zero force?

A. Zero. There is: $m = F/a$. So for $a = 0$ we have: $m = F/0 = 0$. Therefore, the photons have a zero inertial mass (not "infinite mass").

2⁰ **Q.** What is the force of gravity between the material points that "fell" (scil. Distance between which is zero: $r = 0$)?

A. This force is obviously zero:

$$F = \frac{GMm}{r^2} = \frac{GMm}{0} = (GMm) * 0 = 0.$$

For gravity is, by definition, a certain "interaction at distance". Therefore, for a non-zero (i.e. some, certain) force of gravity to exist, there must also be a non-zero (i.e. some, certain) distance between the interacting objects.

3⁰ **Q.** What is the Hamiltonian (energy) perturbation for resonance orbits?

A. It is zero (the disturbance disappears for resonances). Because if

$$S(\vec{J}, \vec{\theta}) = \vec{\theta} \cdot \vec{J} + i\varepsilon \sum_{K \neq 0} \frac{H_{1,K}(\vec{J})}{\vec{K} \cdot \vec{\omega}(\vec{J})} e^{i\vec{K} \cdot \vec{\theta}},$$

thus for $\vec{K} \cdot \vec{\omega}(\vec{J}) = 0$ we receive

²³ Cf. Kaplan, R. *The Nothing That Is. A Natural History of Zero*, Oxford University Press, 2000: "zero is a number that doesn't number".

²⁴ Saitoh, S. *Introduction to the Division by Zero Calculus*, Scientific Research Publishing, Inc. January 1, 2021.

$$i\varepsilon \sum_K \frac{H_{1,K}(\vec{J})}{\vec{K} \cdot \vec{\omega}(\vec{J})} e^{i\vec{K} \cdot \vec{\theta}} = 0,$$

and

$$S(\vec{J}, \vec{\theta}) = \vec{\theta} \cdot \vec{J}.$$

This is consistent with empirical evidence, i.e. with observational stability of astronomical resonances in our Solar System²⁵ as well as in other planetary systems²⁶.

4⁰ **Q.** What form does the formula for the Lorentz transformation from one coordinate system (x, y, z, t) to another (x', y', z', t') for the motion with speed $v = c$ in the direction of x ?

A. If we take the logically necessary zero division result by zero, then we get:

$$\begin{aligned} x' &= 0 \\ y' &= y \\ z' &= z \\ t' &= 0. \end{aligned}$$

Thus, we will receive only the *de facto* already accepted in physics conclusion that for a lux moving at a speed of light there is a maximum shortening ("collapse") of length in the direction of motion [$x' = 0$] and "time stops" [$t' = 0$]. This corresponds to the physical intuition that "time does not flow for light". There is no classically understood motion in the inertial system associated with the photon.

5⁰ **Q.** What is the density of matter and the curvature of space-time in the middle of a black hole?

A. Both of these values reach zero value there. Substituting $r = 0$ in formula for the curvature s of space-time

$$ds^2 = -c^2 \left(1 - \frac{2Gm}{c^2 r}\right) dt^2 + \left(1 - \frac{2Gm}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega$$

we get a flat (and not an illogical: "infinitely curved") spacetime in the centre of a black hole. Also, the density of matter for $r = 0$, i.e. at a point of zero volume must be equal to zero. The question arises what matter is when its density becomes zero. However, the answer will be placed in the last part of this paper.

6⁰ **Q.** What is the density of (electric) point charge?

A. That density is zero. By definition, the density of a given quantity is the ratio of that quantity to the spatial volume in which it is contained. Therefore, if there is a lack of spatial volume (the spatial volume is of zero amount, i.e. there is none of it) then there is no such ratio (this ratio is of zero amount, i.e. it is zero). Thus, the elementary electric charge does not have spatial density and - in this sense - it is not a spatial (extended scale) entity, but only (at most) temporal. I will elaborate on this in detail in subsequent parts of this work.

It can be seen from the above examples that the removal of abstract, logical thinking from science led to the degeneration of science, so that it ceased to be a science, and became only a kind of mindless technique (as Abraham Maslow said, modern science is a kind of technique that enables creative activities of uncreative people²⁷).

²⁵ Molchanov, A. M. (1966). Dokl. Akad. Nauk SSSR 168, 1966.

²⁶ Hadjidemetriou, J. D. "On periodic orbits and resonance in extrasolar planetary systems", *Celestial Mechanics and Dynamical Astronomy*, September 2008, Volume 102, Issue 1–3, pp 69–82.

²⁷ Maslow, A.H. *The Psychology of Science: A Reconnaissance*, New York : Harper & Row, 1966.

3. The Unified Field Theory

As the author notes, some people dealing with physics, mathematics, or their popularization have been claiming for some time that the main problem standing in the way of the Unification of electromagnetism and gravity is the lack of a logically relevant concept of zero and division by zero in mathematics. As Charles Seife stated in one of his well-known books:

„General relativity and quantum mechanics were bound to be incompatible. The universe of general relativity is a smooth rubber sheet. It is continuous and flowing, never sharp, never pointy. Quantum mechanics, on the other hand, describes a jerky and discontinuous universe. What the two theories have in common — and what they clash over — is zero”²⁸.

Based on the results from the previous paragraph, Krzysztof Zawisza introduces a formal hypercomplex algebra with division by zero in mathematics, identifying the number zero with the set of real and (hyper)complex units and introducing arithmetic operations on numbers as a special case of operations on sets of numbers.

For this purpose, constructions such as **the one dimensional (hyper)complex number line** are introduced in the work. The Author begins by logically proving the following identity

$$\infty^{-1} = -\infty. \quad (6a)$$

The above identity results immediately from the logical analysis of the relevant concepts. For since the (potentially) infinitely large quantity is infinitely greater than any finite quantity, then the (potentially) infinitely small quantity is infinitely smaller than any finite quantity. In general, infinitely greater means both "infinitely many times greater" and "greater by infinite quantity". Likewise, infinitely smaller means both "infinitely many times smaller" and "less by infinite magnitude". By denoting the class of infinitely large magnitudes by **IG** and infinitely small ones by **IS**, we get

$$I_G \in IG \leftrightarrow \forall r, t \in R: (I_G > r \cdot t) \& (I_G > r + t).$$

$$I_S \in IS \leftrightarrow \forall r, t \in R: (I_S < r/t) \& (I_S < r - t).$$

Any infinitely small quantity must be arbitrarily many times smaller than any real number ($I_S < r/t$, where r, t - any), but at the same time the infinitely small quantity must be smaller than any real number minus any real number ($I_S < r - t$; r, t - any). Which leads straight to the equation (6a).

The identity of the value of ∞^{-1} and $-\infty$ in an extended set of real numbers results directly from *the principle of identity of indiscernibles*: Since the inverse of ∞^{-1} of the infinitely large quantity ∞ (i.e. the inverse of the quantity greater than any finite quantity) is the infinitely small quantity, so this inverse is a quantity smaller than any finite quantity, i.e. $-\infty$. Assuming now the applicability of basic arithmetic operations to infinite numerical quantities, i.e. extending the domain of basic arithmetic operations to infinite extension of finite sets, we obtain the following *pre-formal notation* of logical necessary implications²⁹:

$$(\infty^{-1} = -\infty) \rightarrow (\infty^2 = -1) \rightarrow (\infty = \sqrt{-1}) \rightarrow ((\infty = i) \vee (\infty = -i)),$$

where ∞ – real potential infinity (simply understood as the limit of a divergent sequence $\{n\}_{n \in \mathbb{N}}$), i – imaginary unit. Thus, anyway

²⁸ Seife, Ch. *Zero. The Biography of a Dangerous Idea*, Penguin, 2000.

²⁹ The necessity to adopt a broader structure in mathematics: an intuitive model, conceptual schemes, etc., is indicated by many mathematicians; cf. *bezogene Existenz* (Bernays 1950), *conceptual schemes, intuitive model* (Mc Naughton 1954, Mc Naughton 1957). See also: Myhill 1952, Lorenzen 1953, Lorenzen 1957, Lorenzen 1975, Hersch 1991, Kreisel 1969. After the discovery of the theory of categories and *topoi*, mathematicians also talk about "local mathematics", contrasting it with the former global tendencies (such as - unrealizable - the postulate of one universal language for all mathematics); see e.g. papers by F. W. Lawver or J. L. Bell (Bell 1986).

$$\infty = \pm i. \tag{6b}$$

Imaginary numbers, as not having all (but having only some) of the features of numbers, are not wholly numbers. They are therefore never complete numbers, and therefore never finite amounts. They are therefore infinite amounts (quantitative infinities). So if $\pm i = \sqrt{-1}$ is a certain infinity, and at the same time $\pm i$ is a unit (scil. It is an imaginary unit), that is, it is a certain unit, then $\pm i$ is a quantitatively infinite real unit. Thus $\pm i$ is an infinite quantum of real unit(s) [*dictum de omni et nullo principium*]:

$$(-i = 1 + 1 + 1 + \dots) \vee (i = 1 + 1 + 1 + \dots). \tag{7a}$$

For important mathematical reasons (see Fig. 2a), we choose the negative sign next to i :

$$-i = \sum_{k=1}^{\infty} k^0 \rightarrow i = -\sum_{k=1}^{\infty} k^0. \tag{7b}$$

From strictly logical postulates the following cyclic order³⁰ in the set of complex numbers, shown in the figure below, follows. According to the understanding of the number zero as a set of four real and imaginary units, we can write a finite complex number line ($B(\mathbb{C}) = \begin{pmatrix} i & -1 \\ 1 & -i \end{pmatrix}$) graphically presented in Fig. 2a. **From the classical, formal point of view, the order of the complex number line is a chain and an anti-chain at the same time.**

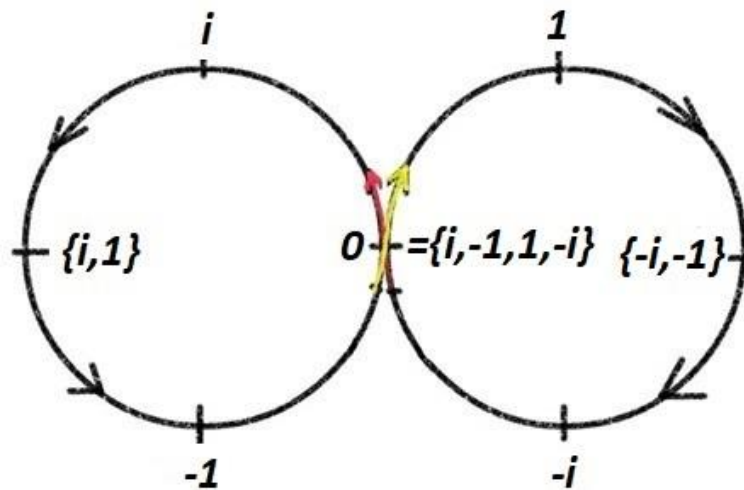


Figure 2a | A finite (i.e. not infinite) complex number line, formed by catenation at infinity of the "end" of the positive real semi-axis with the "beginning" of the negative imaginary semi-axis and the "end" of the positive imaginary semi-axis with the "beginning" of the real negative semi-axis. Infinitely large real values "continuously" evolve into infinitely small (infinite negative) imaginary values, and infinitely large imaginary values pass continuously into infinitely small real values (into infinitely negatives). *The set zero O* (as a set of four unit vectors $\{i, -1, 1, -i\}$) spans the set of complex numbers understood as a vector space over \mathbb{O} .

Historical parenthesis

According to Leonhard Euler the value of the expression

$$\frac{1}{-1} = 1 + 2 + 3 + 4 + \dots$$

³⁰ Huntington, E.V., "Sets of Completely Independent Postulates for Cyclic Order", *Proceedings of the National Academy of Sciences of the United States of America*, 10 (2), 15 February 1924: pp. 74–78.

is **greater than infinity**. Also according to the Swiss mathematician, in the following sequence of fractions:

$$\frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{1}{1}, \frac{1}{0}, \frac{1}{-1}, \frac{1}{-2}, \frac{1}{-3}, \dots$$

the first four terms are seen to grow, then grow to infinity, and beyond infinity they become negative³¹.

In the above Euler's observations there is a prefigure of the cyclic understanding of order in the infinite sets of classical numbers.

The formal definition

In order to formally define *the finite complex number line* C^f based on the above insights, we introduce C^f as the sum of two projective lines C_1P^1 and C_2P^1 .

Postulate

$$1^0 0 \in C_1P^1 \rightarrow 0^{-1} = \{-1, -i\}.$$

$$2^0 0 \in C_2P^1 \rightarrow 0^{-1} = \{1, i\}.$$

One can save this in brief

$$1^{0'} 0_{C_1P^1}^{-1} = \{-1, -i\},$$

$$2^{0'} 0_{C_2P^1}^{-1} = \{1, i\}.$$

Thus we can identify

$$3^0 0_{C_1P^1 \cup C_2P^1}^{-1} \equiv \{-1, -i\} \cup \{1, i\} = \{1, -i, i, -1\}.$$

Definition 3

The *Finite Complex Number Line* C^f is the union of two above projective lines

$$C^f = C_1P^1 \cup C_2P^1,$$

where C_1P^1 and C_2P^1 are determined as above \square .³²

The finite complex number line (**one-dimensional complex axis**) determined above is homeomorphic to a lemniscate and it is **not** – unlike the real projective line – a differential manifold, as it's got an intersection point (point 0) that is not locally homomorphic to the one-dimensional Euclidean space.

³¹ Euler, L. *Vollständige Anleitung zur Algebra*, St.-Petersburg, 1770. English translation (from French) by John Hewlett in: *Elements of Algebra by Leonard Euler* (with the Notes of M. Bernoulli), London, 1822, https://books.google.pl/books?id=X8yv0sj4_1YC&redir_esc=y.

³² The set parts of the finite complex number line: C_1P^1 and C_2P^1 should not be confused with the classic Riemannian complex projective line, which is a manifold, topologically equivalent to the S^2 sphere.

Analogically to the lexicographic order from Fig. 2a the Author introduces a similar structural order in the set of quaternions³³, octonions and for all kinds of hypercomplex numbers of the Cayley–Dickson construction³⁴.

In the set of quaternions³⁵ we get(cf. Fig. 2b):

$$\forall r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8 \in R_+ : r_1 k < -r_2 j < r_3 i < -r_4 < r_5 < -r_6 i < r_7 j < -r_8 k.$$

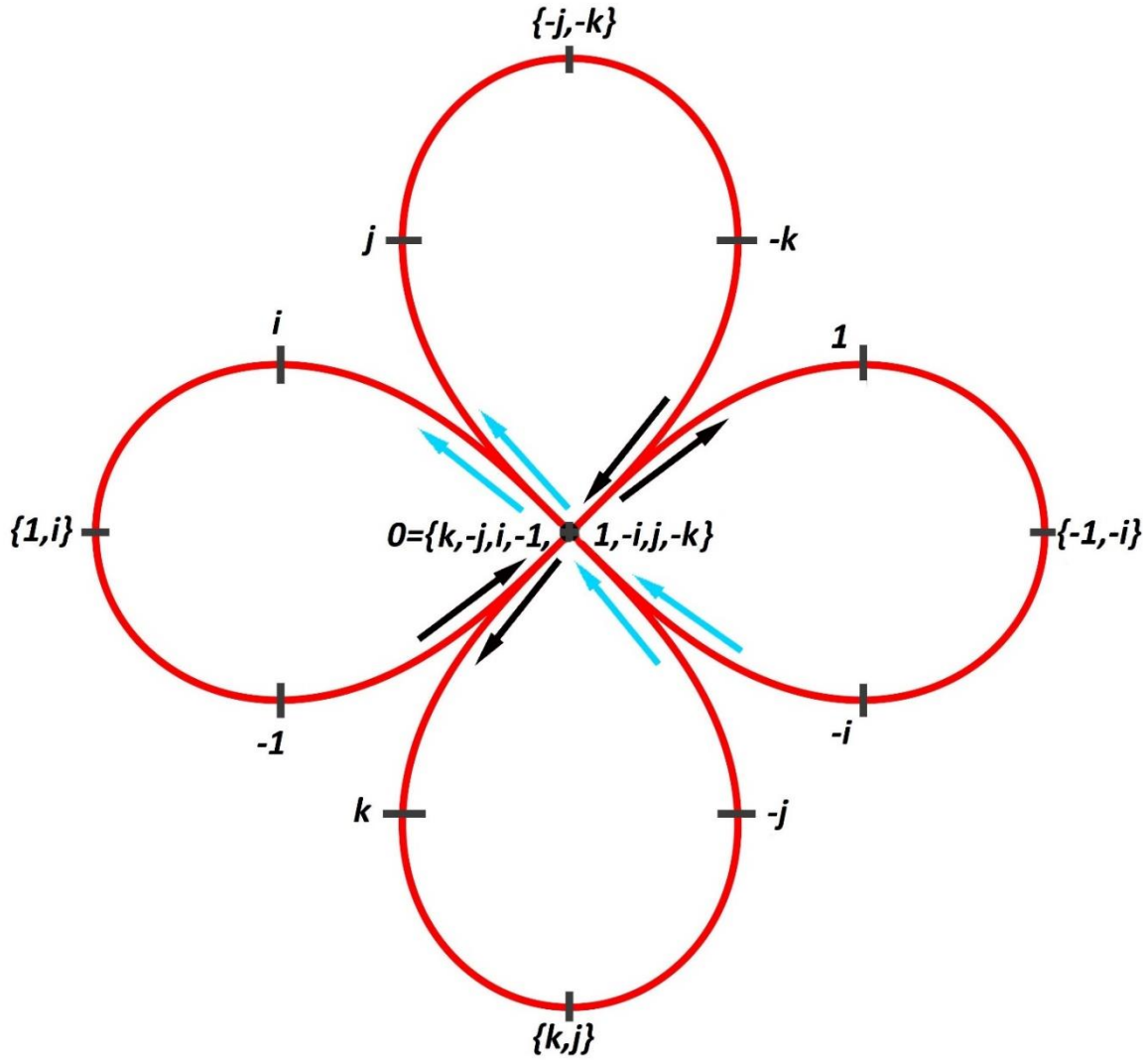


Figure 2b | 1-dimensional *quaternion number line*, constructed analogically to the *complex number line* from Fig. 2a. The lexicographic order within it is chosen by starting from the point *O* counter clockwise directly to the imaginary unit number *k*.

The *quaternion number line* is the set of all quaternion numbers **H** ordered in a *cyclic order* given by the *octonary relation* $[-1, 1, -i, j, -k, k, -j, i]$ and the relevant lexicographic order.

³³ Cf. Hamilton, W.R. "Theory of Quaternions", *Proceedings of the Royal Irish Academy*, Vol. 3 (1844 - 1847), pp. 1-16.

³⁴ Dickson, L. E., "On Quaternions and Their Generalization and the History of the Eight Square Theorem", *Annals of Mathematics*, Second Series, 20 (3), 1919: 155–171.

³⁵ Cf. Hamilton, W.R. op. cit.

The **quaternion number line** can be treated as the set sum of four relevant projective lines [cf. the previous definition of *complex number line*].

We can construct hypercomplex number lines analogously for all kinds of hypercomplex numbers from the Cayley–Dickson construction³⁶ [Cf. Fig. 2c].

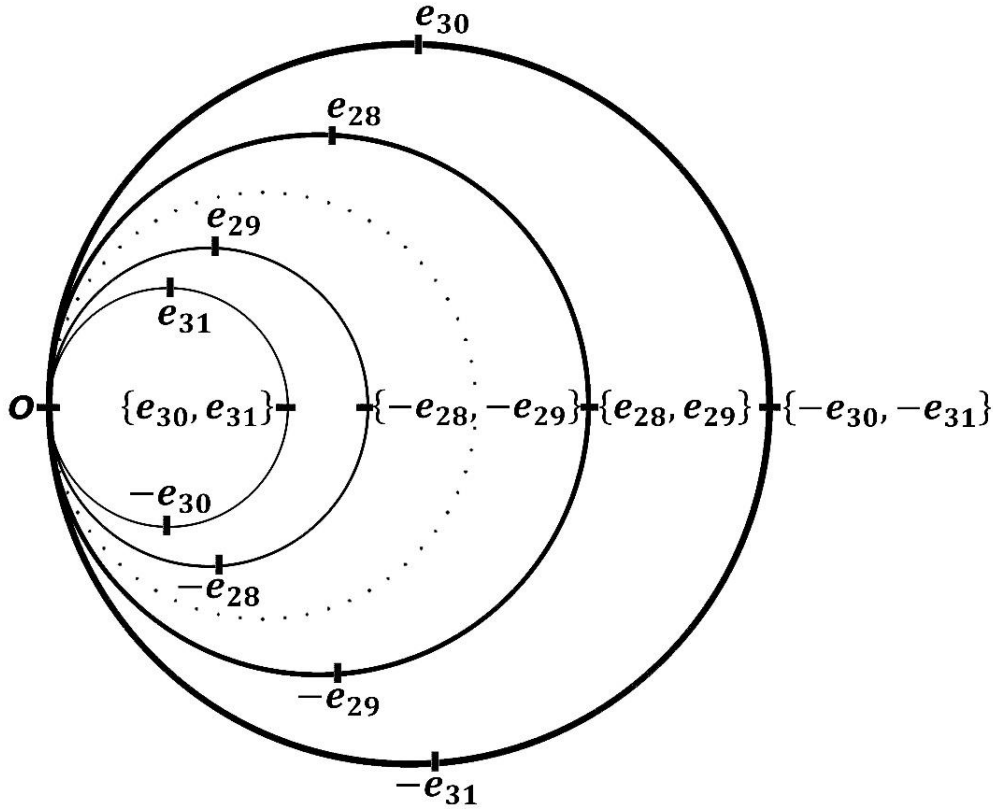


Figure 2c | Alternative hypercomplex number line scheme exemplified by trigintaduonion³⁷ number line. The above ordering of the 1-dimensional trigintaduonion number line is a cyclic clockwise order consecutively ordered from a smaller circle to a greater one, with the smallest circle as *greater than the greatest*. $\bar{O} = (e_{31}, -e_{30}, \dots, -e_0, e_0, \dots, e_{30}, -e_{31})$.

We can take the number zero as the beginning of the number line, that is, as what is "absolutely prime", i.e. what is initial. From a purely logical point of view, it will then be unity as such. Unity (as quality) is the same as continuity, sequence, and therefore – an **order**. Such a vision of zero implies perceiving it as a well-ordered set of complex units (cf. Fig. 2a):

$$\bar{O} = (\{i, -1, 1, -i\}; <), \tag{8a}$$

where " $<$ " = " $i < -1 < 1 < -i$ ".

The set zero $O \in \bar{O}$ can be treated as a set sum of the *real zero* O_R and the *imaginary zero* O_I :

$$O = \{-1, 1, -i, i\} = \{-1, 1\} \cup \{-i, i\} = O_R \cup O_I, \tag{8b}$$

where $O_R = \sqrt{1}$, $O_I = \sqrt{-1}$.

³⁶ Dickson, L. E., "On Quaternions and Their Generalization and the History of the Eight Square Theorem", *Annals of Mathematics*, Second Series, 20 (3), 1919: 155–171.

³⁷ Cawagas, R.E. et al., "The Subalgebra Structure of the Cayley-Dickson Algebra of Dimension 32 (trigintaduonion)", <https://arxiv.org/abs/0907.2047>.

This can be abbreviated as

$$\boxed{0 = \sqrt{\pm 1}} \quad (8c)$$

The formal structure

This part begins with the following definition

Definition 4

Mathematical objects U and V are *arithmetically equivalent* ($U \stackrel{A}{\equiv} V$) iff they are interchangeable (*salvis consequitur*) in all arithmetic operations.

$\forall U, V, W$ – mathematical objects, $\forall \circ$ - an arithmetic operation with U, V, W in its domain:

$$(U \stackrel{A}{\equiv} V) \Leftrightarrow \forall W: (U \circ W = V \circ W) \ \& \ (W \circ U = W \circ V).$$

Then the author formalizes three *logical archetypes* (i.e. logically necessary, basic intuitions) and presents the most important logical conclusions flowing from them.

Archetype A1

$$\forall a \neq \emptyset: \{a\} \equiv a.$$

Conceptual interpretation. Operationally, any individual is its own set [this follows immediately from the *principium identitatis indiscernibilium*: if I've got a set consists of one apple, I've got simply one apple]. It is easy to show that taking this archetype as a thesis immediately **solves the Cantor Paradox**.

Conclusion 1

$$\forall a \neq \emptyset: a \in a.$$

Conceptual interpretation. Everything that is not non-being is itself [*principium identitatis*]³⁸.

Archetype A2

$$\{0\} \equiv \emptyset.$$

Conceptual interpretation. A set containing only *zero* is a set containing only *nothing*, i.e. it is a set containing nothing; hence it is an empty set [*the principle of the non-contradiction of being*].

Conclusion 2

$$\forall A - \text{set}: A \equiv A \cup \{0\}.$$

That's an immediate conclusion from Archetype 2 and from the classical set equality:

$$\forall A - \text{set}: A = A \cup \emptyset.$$

Conceptual interpretation. Adding number zero to numerical elements of any set is adding numerically nothing, so it is not adding anything at all and it does not change (the content of) that set [*diairesis*].

³⁸ In other words: every being contains in itself; every individual being belongs to itself individually; any individual is of itself, from itself, by itself and about itself.

Conclusion 3

$$\forall A, B; B \subset A : \{A, B\} \equiv A.$$

Then the Author constructs two subalgebras of the complex algebra with division by zero: $\mathcal{C}^I/0$, defined for not well-ordered subsets of the set of complex numbers, and $\mathcal{C}^{II}/0$, defined for all and only well-ordered subsets of the set of complex numbers.

The $\mathcal{C}^I/0$ algebra is defined as follows.

Definition 5

We will call the generalized complex algebra with division by zero the following triplet

$\mathcal{C}/0|^G = (P(C), O, S_A)$; where $P(C)$ – the Power set of complex numbers; $P(C) \ni O = \{-1, 1, -i, i\}$ – item singled out from the set $P(C)$ („the set zero”); $S_A = \{\circ: P(C) \times P(C) \ni (Z_1, Z_2) \xrightarrow{\circ} Z_3 \in P(C)\}$ – the set of arithmetic operations in the set $P(C)$, which we will set up next.

Definition 6

Let „ $<_o$ ” be some relation of *linear order* in \mathcal{C} (lexicographic order); let X, Y be at most countable subsets of the set of complex numbers ($X, Y \in P(C)$) with the above relation „ $<_o$ ” defined in them, which generates a *linear order* in them. Let $(X, <_o) = \{x_1, x_2, \dots, x_k, \dots\}$, $(Y, <_o) = \{y_1, y_2, \dots, y_l, \dots\}$.

Let $X, Y \subset C, X = \{x_1, x_2, \dots, x_k, \dots\}, Y = \{y_1, y_2, \dots, y_l, \dots\}$ such that $x_1 < x_2 < \dots < x_k < \dots; y_1 < y_2 < \dots < y_l < \dots$, then

$$X \overset{S}{+} Y = \{x_1 + y_1, x_2 + y_2, \dots, x_k + y_k, \dots\}. \quad (9a)$$

Note 1

If $X = \{x_1, x_2, \dots, x_k\}, Y = \{y_1, y_2, \dots, y_l\}, k < l$, then we use the logically necessary equivalence **A2**:

$\{0\} \equiv \emptyset$ [a set of zero content is a set of no content]. Thus

$$X = \{x_1, x_2, \dots, x_k\} = \left\{ x_1, x_2, \dots, x_k, \underbrace{0, 0, \dots, 0}_{(l-k) \times 0} \right\}.$$

$$X \overset{S}{-} Y = \{x_1 - y_1, x_2 - y_2, \dots, x_k - y_k, \dots\} \quad (9b)$$

$$X \overset{S}{\cdot} Y = \{x_m y_n : x_m \in X, y_n \in Y\} \quad (9c)$$

$$X \overset{S}{/} Y = \{x_m / y_n : x_m \in X, y_n \in Y\} \quad (9d)$$

$$\forall k \in N: X^{(S)k} = \underbrace{X \cdot X \cdot \dots \cdot X}_{k \times X} = \underbrace{(\dots (X \cdot X) \cdot \dots)}_{k \times X} \cdot X \quad (9e)$$

$$\forall q \in Q (q = k/l): X^{(S)k} = \underbrace{(X \cdot X \cdot \dots \cdot X)}_{k \times X} / \underbrace{(X \cdot X \cdot \dots \cdot X)}_{l \times X} \quad (9f)$$

$$\forall r \in R: X^r = \lim_{n \rightarrow \infty} (X^{q_n}), \quad (9g)$$

where q_n – n -reduction of continued fraction representation of r .

The above definitions can easily be generalized to uncountable subsets of \mathcal{C} .

Definition 7'

Let $C \supseteq A = \{a_r \in C: r \in R\}$, $C \supseteq B = \{b_s \in C: s \in R\}$ and there exists a bijection $\mathfrak{S}: A \leftrightarrow B$ that maintains the ' $<$ ' order, i.e. such bijection that

$$\forall t, u \in R, t \neq u: ((a_t, b_t) \in \mathfrak{S} \& (a_u, b_u) \in \mathfrak{S}) \rightarrow (((a_t < a_u) \& (b_t < b_u)) \vee ((a_t > a_u) \& (b_t > b_u))).$$

Then the arithmetic sum (difference etc.) of sets A and B due to the bijection of \mathfrak{S} , will be

$$A \overset{\mathfrak{S}}{+} B = \{a_v + b_v: v \in R, (a_v, b_v) \in \mathfrak{S}\}, \quad (9a')$$

$$A \overset{\mathfrak{S}}{-} B = \{a_v - b_v: v \in R, (a_v, b_v) \in \mathfrak{S}\}, \quad (9b')$$

$$A \overset{\mathfrak{S}}{\cdot} B = \{a_v b_w: v, w \in R, a_v \in A, b_w \in B\}, \quad (9c')$$

$$A \overset{\mathfrak{S}}{/} B = \{a_v / b_w: v, w \in R, a_v \in A, b_w \in B\}, \quad (9d')$$

$$A^B = \{A^{b_w}: b_w \in B\}. \quad (9e')$$

In view of the latter we have got:

$$\text{Log}_X Y = \{Z \subset C: X^Z = Y\}. \quad (9e'')$$

Example 1

Let $A = \{x \in R: 1 \leq x \leq 2\}$, $B = \{x \in iR: \frac{1}{2}i \leq x \leq i\}$, $\mathfrak{S}: x \rightarrow \frac{1}{2}ix$, then

$$A \overset{\mathfrak{S}}{+} B = \left\{x + \frac{1}{2}ix: x \in [1, 2]\right\}.$$

Definition 8

A contracted complex algebra with division by zero (or simply - a complex algebra with division by zero) $C/O|A$ is a generalized complex algebra with division by zero, truncated to a set of one-element subsets of the set of complex numbers (i.e., according to *Archetype 1*, to the set of complex numbers) and the element O (the set zero).

$$\forall X \in C/O|G: X \in C/O \leftrightarrow \exists x \in C: (X = \{x\}) \vee (X = O).$$

Therefore:

$$C/O|A = (C, O, S_A); \text{ where } C - \text{the set of complex numbers; } P(C) \ni O = \{-1, 1, -i, i\};$$

$S_A = \left\{ \circ: P(C) \times P(C) \ni (Z_1, Z_2) \xrightarrow{\circ} Z_3 \in P(C) \right\}$ - the set of arithmetic operations in the set $P(C)$ defined above.

On the basis of complex algebra with division by zero, the basics of a new, logically consistent complex algebra in mathematics and the basics of alternative mathematical analysis is introduced.

Fact 1

$$\forall r \in R: O^r = O. \quad (9h)$$

This is the direct conclusion from (9g) and from the definition of the set O .

Conclusion 3

It is therefore

$$0^0 = 0. \quad (9i)$$

Fact 2

$$\forall x \in O: x \cdot O = \{x\} \cdot O = O. \quad (9j)$$

Fact 3

$$\forall x \in C, x \notin O: x/O = x \cdot O = \{x\} \cdot O = \{-x, x, -ix, ix\}. \quad (9j')$$

Conclusion 4

$$\forall x \in C, x \notin O: (x/O)/x = (x \cdot O)/x = O = (x \cdot O)/(-x) = (x/O)/(-x).$$

Thus, division/multiplication by zero within complex algebra with division by zero is invertible up to the sign.

From the previous definitions and findings, two more important conclusions emerge.

Conclusion 5

The set of complex numbers C with arithmetic operations on the sets defined above (truncated to the set of one-element subsets of C and the set O) is an algebraic field [scil. **it is a field of complex numbers with division by zero**; abbreviated as C/O].

Conclusion 6

The set $O \in P(C)$ with arithmetic operations as above is a sub-field of the algebraic field C/O . **It is therefore an example of a field with one element.**

Examples*Example 2*

From the **Note 1** we have

$$\frac{0}{0} \equiv \{0, 1\}.$$

On the ground of the algebra presented here, this result is logically consistent and logically necessary, because we have

$$\{0, 1\} \equiv \{O, 1\} = \{\{i, -1, 1, i\}, 1\} = \{i, -1, 1, i, 1\} = \{i, -1, 1, i\} = O \equiv 0.$$

Example 3

$$\forall z_1, z_2, \dots, z_k \in C: \sin(\{z_1, z_2, \dots, z_k\}) = \{\sin(z_1), \sin(z_2), \dots, \sin(z_k)\},$$

$$\cos(\{z_1, z_2, \dots, z_k\}) = \{\cos(z_1), \cos(z_2), \dots, \cos(z_k)\}.$$

Sine and cosine on the left side of the equations above are not functions of many variables, but of one multiple variable.

Example 4

Let's calculate the complex polyadic value of the function $f(x) = \log x, x \in C$ for $x = 0$.

Given that $\log 0 = \log 0^{-1} = -\log 0$, we have

$$\log 0 = [\log\{-1, 1, -i, i\}] \cup [-\log\{-1, 1, -i, i\}]$$

Wherein $\log 0 = \log\{-1, 1, -i, i\} = \{i(2\pi n + \pi), 2i\pi n, \frac{1}{2}i(4\pi n - \pi), \frac{1}{2}i(4\pi n + \pi)\}, n \in \mathbb{Z}$.

For $n = 0$ we've got $\log\{-1, 1, -i, i\} = \{i\pi, 0, -\frac{1}{2}i\pi, \frac{1}{2}i\pi\}$.

In conclusion $\log 0 \equiv \log 0 = \{-i\pi, i\pi, 0, -\frac{1}{2}i\pi, \frac{1}{2}i\pi\}$.

The sum AV and the product AP of all elements of the set above are zero

$$AV(\log 0) = 0; AP(\log 0) = 0.$$

Therefore

$$AP(\log 0) \in \log 0.$$

Thus, in general, the general value of the function $f(x) = \log x$ for $x = 0$ is 0:

$$\boxed{\log 0 = 0},$$

while the particular values of the logarithmic function in the point 0 are:

$$-i\pi, i\pi, -\frac{1}{2}i\pi, \frac{1}{2}i\pi.$$

Example 4'

Now let's calculate the value of 0^i . We will use Euler's formula. Given that $0 = 0^{-1}$ we have

$$0^i = \{\cos(\log 0) + i \sin(\log 0)\} \cup \{\cos(\log 0) - i \sin(\log 0)\}.$$

Now using the Examples 2, 3 and the formula (9a) we get

$$0^i = \{e^\pi, e^{-\pi}, 1, e^{\pi/2}, e^{-\pi/2}\}.$$

There is also

$$0^{-i} = 0^i = \{e^\pi, e^{-\pi}, 1, e^{-\pi/2}, e^{\pi/2}\}.$$

Since " $-i$ " is the simplest understood (i.e. unitary) infinity, the above equality gives five acceptable values of the simplest (i.e. most generally) understood indefinite symbol 0^∞ , the common value of which is

$$AP(0^i) = AP(0^{-i}) = 1.$$

If we define 0^∞ as

$$0^\infty = \underbrace{0 \cdot 0 \dots 0 \dots}_{(1+1+\dots) \times 0 = (-i) \times 0},$$

we can then accept

$$0^\infty = 0^{-i} = 1 = 0^i = 0^{-\infty}.$$

This allows us to determine the value of the indeterminate symbol $0^{\pm i}$, i.e. to valueate also the numerical value of the symbol $0^{\pm \infty}$.

Note 3

Since $0 \cdot (\pm i) = 0$, then we can also assume

$$0 \cdot (\pm \infty) = 0.$$

This allows us to determinate some mathematical indeterminate forms.

In the second complex algebra with division by zero $\mathcal{C}^H/0$ we conclude the division of a complex number by zero as follows

$$\forall x \in \mathcal{C}: \left(\left(\frac{x}{0} = 0 \right) \leftrightarrow (x \neq 0) \right) \& \left(\frac{0}{0} = 1 \right).$$

Example 5

$$\forall k \in \mathbb{N}: 0^k = 0 \leftrightarrow k = 2n - 1; 0^k = 0_R \leftrightarrow k = 2n.$$

Conclusion 7

The classical first derivative of the complex function $f(x)$ in the complex algebra with division by zero $\mathcal{C}^H/0$ will be calculated as follows

$$f'(x) = \frac{f(x+0) - f(x)}{0}. \quad (10a)$$

Conclusion 6 (classical second derivative in the *second complex algebra with division by zero*)

$$\begin{aligned} f''(x) &= \frac{f'(x+0) - f'(0)}{0} = \frac{\frac{f(x+0+0) - f(x+0)}{0} - \frac{f(x+0) - f(x)}{0}}{0} = \\ &= \frac{\frac{f(x+0+0) - f(x+0)}{0} \cdot \frac{f(x+0) - f(x)}{-0}}{0} = \frac{f(x+0+0) - f(x)}{0^2}. \end{aligned}$$

The above relationship, using the fact that 0 is a neutral element of addition, we can rewrite in the following form:

$$f''(x) = \frac{f(x+0+0) - f(x)}{0^2} = \frac{\frac{f((x+0)+0) - f(x+0)}{0}}{0} = \frac{f'(x+0)}{0}. \quad (10b)$$

The complex analysis with division by zero generally simplifies the calculation of derivatives.

Further applications

Analogously one can show that

$$\frac{\partial^2 f(x)}{\partial x^2} = \frac{\partial f(x+0)}{\partial x} \cdot 0 = \frac{\partial f(x+0)}{\partial x} \cdot 0^{-1}. \quad (11)$$

The above relation will enable, among others, the transformation of linear QM equalities into nonlinear GR equations.

Applications in Physics

a) Renormalization

A problematic (according to the physicists themselves) *ad hoc* method to avoid alleged infinities (usually resulting from erroneous concept of division by zero)³⁹. Below I will show with a few simple examples how you can - applying the complex algebra with division by zero - avoid using this compromised method and perform "renormalization without renormalization".

1) Self-interactions in classical physics

It is well known that the mass of a charged particle should include the mass-energy in its electrostatic field. Assume that the particle is a charged spherical shell of radius r_e . The energy in the field is

$$m_{em} = \int dV \frac{1}{2} E^2 = \int_{r=r_e}^{r=\infty} 4\pi r^2 \frac{1}{2} \left(\frac{q}{4\pi r^2} \right)^2 dr = \frac{q^2}{8\pi r_e} \quad (12.a)$$

and in classical physics the last quotient is infinite when r_e is zero, which implies that the point particle would have "infinite inertia". In classical physics to avoid dividing by zero, which results in – as is wrongly, in fact, believed – the particle's infinite mass, so called *renormalization* is performed that consists in abstaining from reaching the zero distance when calculating the mass and charge, and stopping at a certain, arbitrarily set, distance instead. Richard P. Feynman has found the renormalization *a rather dippy process*. The arbitrarily placed value of r_e that makes m_{em} equal to the electron mass is called the *classical electron radius*

$$r_e = \frac{q^2}{4\pi\epsilon_0 m_e c^2} \approx \frac{1}{137} \frac{\hbar}{m_e c} \approx 2.8 \cdot 10^{-15} m.$$

Finite value of the electron radius stands in contradiction with the necessary assumption, saying that elementary charges must be the size of a point. However, in mathematics with division by zero, we have from (12.a)

$$m_{em} = \frac{q^2}{8\pi} \cdot \frac{1}{r} \Big|_{r_\infty}^{r_e}. \quad (12.b)$$

Assuming that $r_e = 0$ and r_∞^{-1} is negligibly small, the finite value of m_{em} is

$$m_{em} = \frac{q^2}{8\pi \cdot (r_e=0)} = \frac{q^2}{8\pi \cdot l_p}. \quad (12.c)$$

This removes the alleged "infinite values" and sets the "natural renormalization distance" to the Planck distance l_p . More precisely, we come to the minimum possible spatial distance, which is the dimension of the point l_p . This removes the conceptual and mathematical difficulties and paradoxes associated with the above issue.

2) Renormalization and the QED

Choosing an increasing energy scale and using the renormalization group makes this clear from simple Feynman diagrams; were this not done, the prediction would be the same, but would arise from complicated high-order cancellations.

³⁹ Cf. Huang, K. "A Critical History of Renormalization", *International Journal of Modern Physics A*, Vol. 28, No. 29, 1330050 (2013).

For example,

$$I = \int_0^a \frac{1}{z} dz - \int_0^b \frac{1}{z} dz = \ln a - \ln b - \ln 0 + \ln 0.$$

Thus, on the ground of *classical analysis* I is ill-defined.

To eliminate the divergence, we usually change lower limit of integral into ε_a and ε_b :

$$I = \ln a - \ln b - \ln \varepsilon_a - \ln \varepsilon_b = \ln \frac{a}{b} - \ln \frac{\varepsilon_a}{\varepsilon_b}.$$

Making sure $\frac{\varepsilon_a}{\varepsilon_b} \rightarrow 1$, then $I = \ln \frac{a}{b}$.

However, using the algebra with division by zero, according to *Example 4*, we can substitute

$$-\ln 0 = \ln 0 = 0.$$

Thus we get the same general result in the simpler way.

b) Basic relationships of quantum mechanics

Heisenberg picture

$$\frac{d}{dt} A(t) = \frac{i}{\hbar} [H, A(t)] + \left(\frac{\partial A}{\partial t} \right).$$

Uncertainty principle

$$[p, x] = px - xp = -i\hbar.$$

Schrödinger picture

$$i\hbar \frac{\partial}{\partial t} \Psi = \hat{H} \Psi.$$

Dirac equation

$$(\beta mc^2 + \sum_{k=1}^3 \alpha_k p_k c) \psi(\vec{x}, t) = i\hbar \frac{\partial \psi(\vec{x}, t)}{\partial t}.$$

They all are linear equations of the form

$$A = iB, \tag{13}$$

where A, B – certain operator expressions, i - imaginary unit. The right part of the equations (13) can be taken as one of the results of multiplying / dividing B by zero (*scilicet*: by the set O), i.e. as one of the results of zeroed B (i.e. **computing the value of expression B at a point**).

This allows for a new interpretation of the quantum mechanics equations and to penetrate deeper into their logical meaning. And thus, also, into their physical sense.

Unification of electromagnetism and gravity

Based on a logical analysis of the concepts of time, space and matter, and on the results of Janusz Drożdżyński's thought experiment⁴⁰, the Author demonstrates the need to replace the five-element Einstein interval for curved space-time

$$ds^2 = dx^2 + dy^2 + dz^2 + dw^2 - c^2 dt^2 \tag{14a}$$

⁴⁰ Drożdżyński, J. „A Revision of the Principle of Relativity”, PHYSICS ESSAYS, 26, 2 (2013).

with the four-dimensional Pythagorean interval

$$s^2 = R^2 = x^2 + y^2 + z^2 + c^2t^2, \quad (14b)$$

where time functions as the curvature of space (w - the scalar curvature):

$$w = ct.$$

To prove the need to change the physics of space-time from the Minkowski interval to the Pythagorean interval, the Author shows that the currently adopted in physics form of space-time interval (Minkowski interval) leads to irremovable paradoxes. This is because it combines in a contradictory (i.e. mutually exclusive) way the concepts of space and time. Let

$$S^2 = c^2\Delta t^2 - \Delta r^2, \Delta r^2 = \Delta x^2 + \Delta y^2 + \Delta z^2, \quad (15a)$$

(with the common understanding of the above symbols) will be the classic *SR* space-time interval (within planar space-time). Then the distance S between two events in space-time is equal to

$$S = \sqrt{c^2\Delta t^2 - \Delta r^2}. \quad (15b)$$

Dividing both sides of the above equation by Δt we get

$$V_{\Delta t} := \frac{S}{\Delta t} = \sqrt{c^2 - \frac{\Delta r^2}{\Delta t^2}}. \quad (15c)$$

Here $V_{\Delta t}$ is the speed at which the given space-time interval S is travelled over time Δt . As can be seen, **this speed is slower, the greater is the spatial distance Δr travelled during that time.**

However, this contradicts the logical content of the notion of *speed*. Because the objects that move the fastest in space-time would be objects that are spatially immobile, i.e. objects that move only in time. In order to remove this inconsistency with the logical understanding of the concept of motion and the speed of motion, the easiest (the most true) way is to replace the Minkowski interval, based on hyperbolic geometry, with the Pythagorean interval, applicable in Euclidean (planar) geometry⁴¹

$$S^2 = c^2\Delta t^2 + \Delta r^2. \quad (16a)$$

The above Pythagorean form of the space-time interval is this time *in line* with the logical understanding of the concept of speed of movement *in linear* time. We have then

$$V_t = \frac{S}{t} = \sqrt{c^2 + \frac{x^2+y^2+z^2}{t^2}}. \quad (16b)$$

Which in turn means that

- The longer the distance traveled in a given time t , the faster the speed of motion as such (i.e. motion in space-time).
- The speed of light is the **minimum** speed in space-time (limiting speed).

⁴¹ The Pythagorean interval appears already in modern space-time theory while introducing the concept of *imaginary time*. Cf. Chen, X. *Three Dimensional Time Theory: to Unify the Principles of Basic Quantum Physics and Relativity*, <https://arxiv.org/abs/quant-ph/0510010>.

- The speed of light is the speed of the movement in time while resting in space ('speed of the passage of absolute time')⁴² or the speed of movement in space while resting in absolute time (speed of the photon's movement).

A graphic illustration of the Pythagorean Space-Time Interval is shown below.

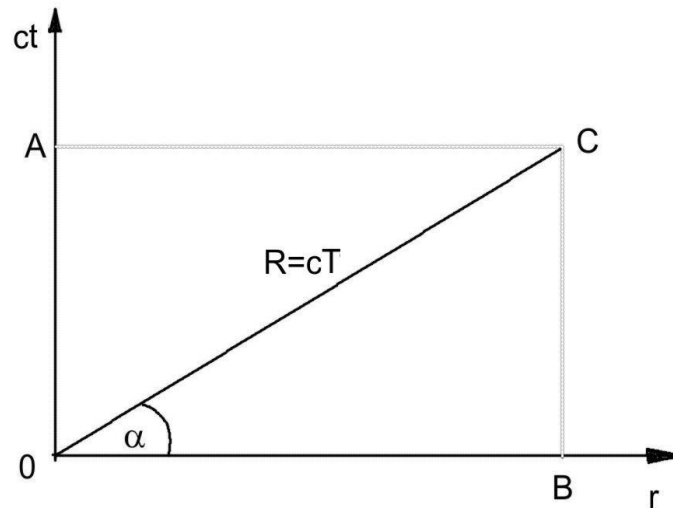


Figure 3 | Logically consistent way to calculate spacetime distances ("intervals"). Here t is the cosmological time (absolute time, i.e. global time) measured from the origin of the coordinate system, r is the distance in space (also counted from the origin of the coordinate system), and T is a relative time (local time). Time T multiplied by the speed of light should be conceptually identified with movement as such: $R = cT$. Every local movement takes place both in (local) place r and in local time T at the same (absolute) time t .

For $r = 0$ we have $cT = ct$ (therefore $T = t$), which means rest in space and moving in time only. The local time is then located in absolute time. In turn, for $t = 0$ we have motion only in space (motion of photon). Then the elapse of local time ΔT is proportional to the distance traveled: $c\Delta T = \Delta r$. Colloquially speaking, everything that happens in a system related to a photon happens - in the absence of cosmological time - due to the movement of the photon in space (and meeting subsequent events in this space). Thus, the space (travelled) in the direction of the photon's motion plays for the photon the role of time.

It can now be easily seen that *the absolute elimination of absolute time t* from physics is a textbook example of logical fallacy – the fallacy in composition⁴³. This error arises when we uncritically ascribe to a certain whole features that only individual (i.e. taken separately) parts of that whole does possess. A textbook example of this textbook error is the following reasoning. All cells are aquatic. Therefore, all organisms (which are composed of cells) are aquatic. This is of course not true. Likewise, since all local (i.e. partial) time is relative, it does not follow that global (cosmological) time is also relative. On the contrary, as essentially Einstein already noted, the existence of absolute cosmological time is the *sine qua non* condition of the existence of relative time. In a reasonable understood reality, parts can exist only as parts of a certain whole. Never in isolation from the latter.

As can also be seen from Figure 3, there is the following relationship

⁴² The speed of light is the speed of information, the speed of change, so it is the speed of consequences of causes and effects. But since the speed of light is the speed of change and the speed of transition from cause to effect, then the speed of light is simply the speed of time.

⁴³ Cf. Bennet B. *Logically Fallacious*, Archieboy Holdings, Boston, 2015.

$$\frac{t}{T} = \sin \alpha = \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - \left(\frac{r/T}{c}\right)^2} = \sqrt{1 - \left(\frac{v}{c}\right)^2}, \quad (17)$$

where $v = \frac{r}{T}$ – relative (local) speed, which is the ratio of the relative distance travelled in space to the amount of relative (local) time in which that travel occurs.

This shows a deeper physical sense of the mathematical form of the Lorentz transformation. The famous Lorentz quantity γ is apparently the local ratio of global time to local time. It is, in other words, the global time (calculated) in terms of local time (global time including relative time), i.e. the global time measured in the local frame of reference.

It can be seen that the Pythagorean interval S is not an invariant of the new theory. This obvious invariant will be the absolute interval $s := ct$. It is a product of two absolute (i.e. both invariant) quantities c and t . From Figure 3 we have

$$s^2 = c^2 \Delta t^2 = \Delta R^2 - \Delta r^2 = c^2 \Delta T^2 - \Delta x^2 - \Delta y^2 - \Delta z^2 \quad (\text{the Minkowski interval}).$$

It can be seen that a properly understood Minkowski interval is the absolute (universal) distance between two events in absolute time t and that is why it is *invariant*. However, it cannot be treated as the distance between events in *properly understood space-time*.

Thus we have:

$$ds = cdT \sqrt{1 - \left(\frac{v}{c}\right)^2}, \quad (18)$$

where $v = \frac{dr}{dT}$.

Let us now take the four-vector $\bar{R} = (cdT, dx, dy, dz)$. Dividing each of the coordinates of \bar{R} by its measure ds , and then multiplying each coordinate of the resulting four-vector by $m_0 c^2$, we obtain (exactly the same way as in the analogous, classical reasoning) the formula for the *relativistic energy*

$$E^2 = E_0^2 + p^2 c^2,$$

where $p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$ – relativistic momentum.

This shows that replacing the Minkowski's interval with the Pythagorean interval does not change most of known physical formulas, but it does change their understanding (i.e. their physical interpretation).

Some consequences and further analysis

The algebra of classical quaternions becomes the model of space-time, which introduces a simple notation for the representation of spatial orientations and rotations⁴⁴. The metric tensor of space-time⁴⁵ takes a simple form

$$[g_{\mu\nu}] \approx [\eta_{\mu\nu}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (19)$$

⁴⁴ Kuipers, J. *Quaternions and Rotations Sequences: A Primer With Applications to Orbits, Aerospace and Virtual Reality*, Princeton Univ. Press, 2002.

⁴⁵ Einstein, A. „Die Grundlage der allgemeinen Relativitätstheorie“, *Annalen der Physik*, 1916 (7), Band 49.

Then, applying the rules of complex algebra with division by zero, we conclude that there is a physical equivalence between mass and charge. To demonstrate this equivalence, the author uses the Pythagorean-Platonic and Leibnitian logical-conceptual analysis, with the help of which science as such was once created and modern science was co-created.

To begin with, we come to the conclusion that time, being a measure of other things, is not yet itself measurable. So it is immeasurable (infinite). And since time is infinite, it never ends and always lasts. This results in durability, i.e. the indestructibility of time, and in its permanence, i.e. in its continuity.

If time is the measure of all things, i.e. the basic measure of the universe, and therefore the universal (basic) proportion. Of course, the most universal (i.e., the simplest, and therefore the most general) proportion is the golden proportion (*aurea sectio*). It should therefore be taken as the basic unit of time, and each time segment must be an integer multiple of this unit. So, we got an archetypal logical identity

$$t_p = \Phi = \frac{1+\sqrt{5}}{2}, \quad (20)$$

where t_p - the Planck time⁴⁶, Φ – the *golden number* [that identity follows directly from the *principium identitatis indiscernibilium*].

The golden number is the simplest (i.e. the “most”) transcendental number

$$\Phi = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}} = \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}$$

According to a known fact in the theory of Diophantine approximations (Borel, 1903), the golden number is (up to a set of measure zero) the number least approximated by rational numbers.

Time, as an irrational quantity, i.e. not measurable number, has no measure. So it is not a dimension (not measurements), but only a measure. At the same time, time, being a scalar amount, not a vector quantity, does not have a definite direction (because in every direction time is never finite), but **it is a direction itself** (scil. time's the direction of change in space). The following statement about a connection between time and *direction* follows *directly* from this:

Statement 1

The simplest (i.e. minimally) understood time (i.e. Planck time) is conceptually equivalent to the concept of direction

$$t_p \equiv " \rightarrow "$$

Considering the fact, that Planck time

$$t_p = \sqrt{\frac{G\hbar}{c^5}} = 5.39106 \times 10^{-44} \text{ s},$$

⁴⁶ Planck, M. "Über irreversible Strahlungsvorgänge", *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin (1899)*, 5: 440–480: «Wählt man nun die »natürlichen Einheiten« so, dass in dem neuen Maasssystem jede der vorstehenden vier Constanten den Werth 1 annimmt, so erhält man [...] als Einheit der Zeit: $\sqrt{\frac{G\hbar}{c^5}} = 1.38 \cdot 10^{-43} \text{ sec}$ » [<https://archive.org/details/sitzungsberichte1899deutsch/page/480/mode/2up>].

we get

$$1 \text{ s} \approx 3.001333 \times 10^{43}.$$

The second, as a unit of time, can therefore be identified with a certain irrational number [as we have established above, time is a number and is also incommensurable, i.e. irrational], i.e. with a certain proportion⁴⁷.

The above conclusion is logically necessary, although it necessarily contradicts common-sense, habitual intuitions, i.e. it denies our fixed associations. These associations (or paradigms of sense) say that physical qualities cannot be reduced to mathematical quantities. However, the Pythagoreans, who have created the science as such, thought differently. For them both the entire Universe and the ontological categories that describe It can ultimately be reduced to numbers⁴⁸. Actually, it is likely that the Pythagoreans were the first to discover that **physical qualities are in fact meta-physical quantities**. These metaphysical, i.e., transcendent quantities are simply infinite numbers - numbers that cannot be wholly represented as a finite number of finite numbers (i.e. finite number of rational numbers)⁴⁹. They are therefore irrational or imaginary numbers

The correctness of identity (20) can be empirically confirmed [or rather empirically *illustrated*, because logically necessary results don't need any empirical confirmation to be considered true]. Let's take the ratios of the lengths of adjacent geological eons, eras and periods. Namely, the golden ratio appears in the basic geological timescale [cf. Fig. 4]. The Paleoproterozoic era began about 2.5 billion years ago; 1.6 billion years ago - this is the beginning of the Mesoproterozoic, 1 billion years ago - the Neoproterozoic, and 0.6 billion years ago there is another turning point - the rapid development of animals.

Cenozoic - the most recent of the three geological eras of the Phanerozoic - is still ongoing; therefore its final length is unknown to us. However, the ratio of the duration of the Palaeozoic to the Mesozoic is: (292 [293] million years) / (186 [185] million years) = 1.57 [1.58]. It is therefore close to the golden ratio [~ 1.62]. Also in particular periods of the Phanerozoic eon, golden relations are visible on many breakthrough dates. The ratio of the duration of the Devonian, Mississippian and Pennsylvanian is (respectively) 55: 34: 20, therefore - within the accuracy of the determination of these periods - equal to the ratio of the successive Fibonacci numbers (in descending order): 55: 34: 21. Also 3 out of the 6 epochs of the Cenozoic Era and the Cretaceous Era, that preceded them, have their origins related to the order of numbers of the Fibonacci sequence - 5, 34, 55 and 144 million years ago [see Fig. 5].

⁴⁷ This result is consistent with the current state of collective consciousness. As noted by the protagonist of the cult film *Pi* by Darren Aronofsky: *Everything could be represented and understood through numbers*.

⁴⁸ Guthrie, W. K. C. *A History of Greek Philosophy*, Vol. 1: *The Earlier Presocratics and the Pythagoreans*, Cambridge, U.K.: Cambridge University Press, 1962.

⁴⁹ Here we use the term "transcendental number" not in its strictly algebraic sense (as opposite to "algebraic number") but in the general, linguistic use.

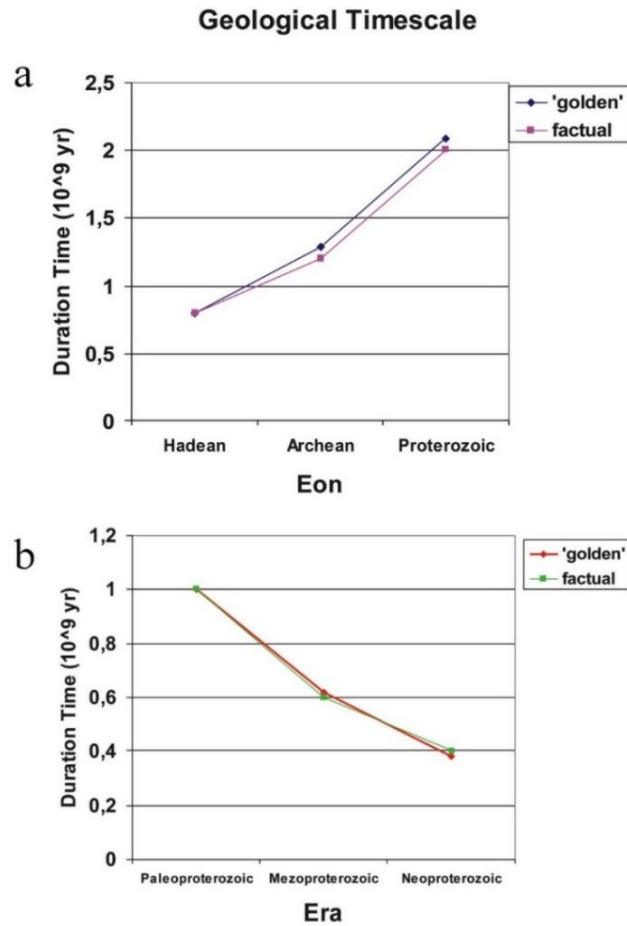


Figure 4 | Duration time ratio of consequent eons and eras in the geological timescale is close to the Golden Mean. **a**, Increasing length of three consecutive eons.: Hadean (4.6-3.8 billion years ago), Archean (3.8-2.6), Proterozoic (2.6-0.6). **b**, Decreasing length of three eras of Proterozoic eon. The differences between the 'golden' and factual values are lesser then the present(ed) accuracy of geological timescale.

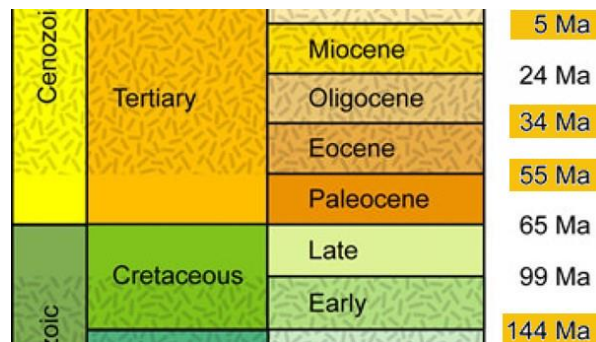


Figure 5 | The clear presence of Fibonacci numbers in dates relating to the beginning of subsequent periods in most recent geological history of the Earth [<https://www.universetoday.com/79271/tertiary-period/>]. It should also be noted that $65/99 \approx 0.66 \approx \Phi^{-1}$; $24/65 \approx 0.37 \approx \Phi^{-2}$; $24/99 \approx 0.24 \approx \Phi^{-3}$.

In order to move from the analysis of the concept of time to the analysis of the concepts of distance speed and velocity, it is necessary to note at the outset that the spatial distance is according to conceptual (logical) definition by stopping (i.e. resetting) or zeroing time. Thus the Planck distance should be defined as Planck time and zero at the same time, or the Planck time time(s) zero:

$$l_p := t_p \cdot 0 \equiv \Phi \cdot \{i, -1, 1, -i\}. \tag{21a}$$

Considering the Statement 1, we can write:

$$t_p \cdot O \equiv \Phi \cdot \{i, -1, 1, -i\} = (\{i, -1, 1, -i\}; <). \quad (21b)$$

Conceptually: the Planck distance (dimension of a point) is generally (i.e. in total) zero. Particularly, however, it is singular [i.e. indivisible].

Otherwise

$$l_p \equiv (\{it_p, -t_p, t_p, -it_p\}; <). \quad (21c)$$

Then the speed of light is

$$c = \frac{l_p}{t_p} = \frac{t_p \cdot O}{t_p} = O.$$

And the velocity of light (speed of light with a direction):

$$\vec{c} = \bar{O} \equiv \Phi \cdot O = \Phi \cdot \{i, -1, 1, -i\} \equiv (\{i, -1, 1, -i\}; <).^{50} \quad (21d)$$

Then, moving from geology to cosmology and from time to length, we have the "golden" structure of space that we actually observe in many distributions of galaxies and dark cold matter.

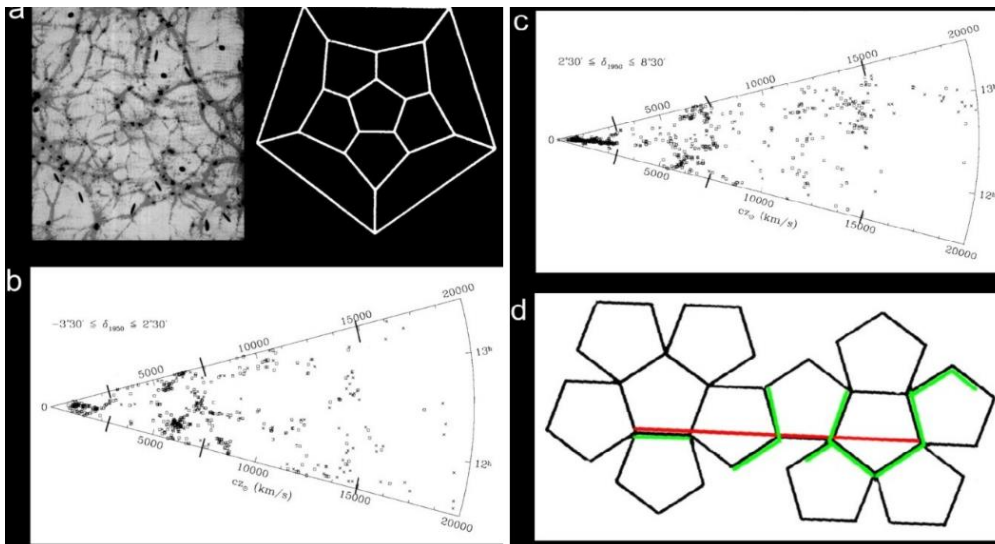


Figure 6] a. Distinct pentagons in one of the CDM distribution diagrams, based on the analysis of image distortions of some of the 170,000 galaxies (bright objects) [left panel; based on: George Musser, „What’s the Matter?“, *Scientific American* 282, 24 (2000)]. As you can see, their distribution resembles the Schlegel diagram of the dodecahedron [right panel], i.e. a regular dodecahedron seen through one of its faces. **b - d.** Comparison of the distribution of galaxies from the *Zwicky Catalog* with the shape and proportions of a regular dodecahedron grid.

⁵⁰ Physical velocity of light c was first time related to mathematical imaginary unit i by Hermann Minkowski: „Führt man dann noch $\sqrt{-1} \cdot t = s$ [...]. Man kann danach das Wesen dieses Postulates mathematisch sehr prägnant in die mystische Formel kleiden: $3 \cdot 10^5 \text{ km} = \sqrt{-1} \text{ sek}$ “ [Minkowski, H., *Raum und Zeit, Jahresberichts des Deutschen Mathematiker-Vereinigung*, Band 18, Verlag von B.G. Teubner, Leipzig und Berlin, 1909].

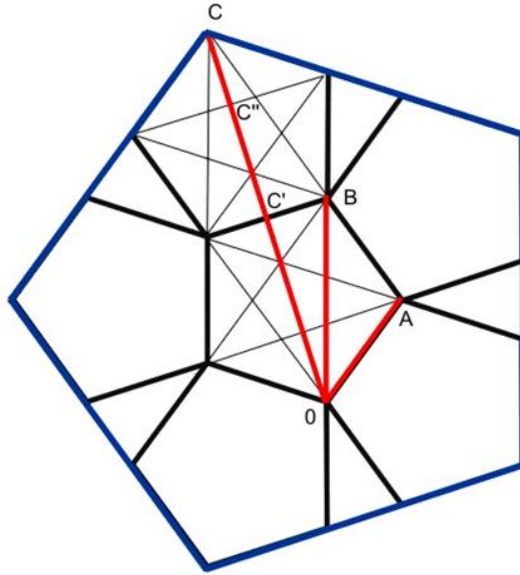


Figure 7 | The geometry of space with time as the curvature of space correlates with the occurrence of the golden ratio in the time relations of the duration of adjacent geological and cosmological epochs. Here: $OA/OB = OB/OC = \Phi^{-1}$; $OC'/C'C'' = C'C''/C''C = \Phi$.

Now we are prepared to construct basics of the Unified Field Theory.

Basic facts

If a physical body is deprived of electric charge (and of nothing else), then (at least) the body's mass (energy) still remains. Therefore, mass (M) may be recognised as "zeroed" charge (Q):

$$\bar{M} = Q/\bar{0}. \quad (22a)$$

In $\mathcal{C}^I/0$ algebra we obtain then

$$Q = \bar{0} \cdot M. \quad (22b)$$

We refer to the $\mathcal{C}^I/0$ algebra and to $\bar{0}$ (zero as an well-ordered set) because electric charge as such (electric charge as a quality) has two signs and thus it is a well-ordered set too: $Q = [-q, q]$.

In other words, the **electric charge is a measure of the density of a mass concentrated at a point**. This explains why in QM the elementary charge must be considered a point charge.

In yet some other words: from the strictly logical point of view, an unified theory of gravitational and electromagnetic impacts has to be founded on a basic and most broadly understood relation (ratio, proportion) between charge and mass. Because charge and mass are *different qualities*, therefore there is no *formal* relation between them (*scil.* the relation is as weak, as possible). In such a case their relationship should be *no relationship* (*scil.* *the relationship* as weak as possible is a *zero relationship*).

Then we obtain

$$Q = \{i, -1, 1, -i; \langle \rangle\} * M.^{51} \quad (22c)$$

⁵¹ Cf. Bondi, H. "Negative Mass in General Relativity", *Reviews of Modern Physics*, Vol. 19, No. 3, July, 1957.

It follows that no electric charge can be attributed to massless objects. The conversion of mass into an electric charge is carried out according to the law (All.C1.a'), and the final products of this conversion are two particles with masses M and $-M$ and electric charges iM and $-iM$, respectively.

From (19d) and (20c) we obtain⁵²

$$Q = M\vec{c}. \quad (22d)$$

Note 4

Electric charge has a size of momentum.

Note 4'

Electric charge can be treated as a momentum of mass moving with the speed of light in [the direction of the arrow of] time⁵³. Therefore, the electric charge has no inertia in space⁵⁴.

Treating an electric charge as a momentum of mass gives rise to, inter alia, the following consequences.

The curvature tensor as a zeroed electromagnetic field tensor

Now we can derive relation between electromagnetic field tensor, its dual tensor and metric tensor of the space-time. *Explicit* form of the electromagnetic field tensor there is as follows

$$F_{\mu\nu} = \begin{pmatrix} 0 & \frac{E_x}{c} & \frac{E_y}{c} & \frac{E_z}{c} \\ -\frac{E_x}{c} & 0 & -B_z & B_y \\ -\frac{E_y}{c} & B_z & 0 & -B_x \\ -\frac{E_z}{c} & -B_y & B_x & 0 \end{pmatrix}. \quad (23a)$$

Let us perceive that the intensity of the electrostatic field changes in space, and the sourceless magnetic field only changes with time. Thus, the electric field is the electromagnetic field the intensity of which is counted per unit of space, and the magnetic field – per unit of time

$$\vec{E}/l_p = \vec{B}/t_p. \quad (24a)$$

⁵² As far as we know Thomas Young (1773 – 1829) was the first who proposed an idea of reducing gravitation to electrostatic force [Young, T. *A course of lectures on natural philosophy and the mechanical arts*, London, 1807]. This idea was than developed, among others, by O.F. Mossotti [*Sur les Forces qui Regissent la Constitution Interieure des Corps*, Turin, 1836], W. Weber [*Elektrodynamische Maassbestimmungen insbesondere über elektrische schwingungen*, S. Hirzel, 1864] and H. A. Lorentz [*Proc. Acad. Sci. Amsterdam* 6,1904]. In 20th century, these attempts recognised gravity and electromagnetism as more often than not two separate impacts which unite in the superior “superforce”. Therefore a search is pursued for a superior description capable of “containing” the two aforementioned forces – each of them separately. Such approach is however a mechanical and external approach only, and the need it implies to complicate existing models and deriving new notions; e.g. “superforce” is inconsistent with Ockham’s principle [Soklakov, A.N. *Occam’s Razor as a formal basis for a physical theory*, arXiv:math-ph/0009007 v3 24 Feb 2002].

⁵³ As Lee Smolin has written, the new unification will assume as its departure point a profound principle, such as the law of inertia or the equivalence principle. This profound and surprising idea will make us realise that the two things that we’ve once seen as unrelated are one and the same thing [Smolin, L. *The Trouble with Physics*, Spin Networks, Ltd. 2006].

⁵⁴ “Does electricity have inertia?”, Posted by u/alat95 6 years ago, https://www.reddit.com/r/askscience/comments/1n64r5/does_electricity_have_inertia/.

Therefore
$$\vec{B} = \frac{\vec{E}}{c}. \quad (24b)$$

Substituting (24b) we obtain from (23a) the dual tensor

$$G^{\mu\nu} = \begin{pmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & -\frac{E_z}{c} & \frac{E_y}{c} \\ -B_y & \frac{E_z}{c} & 0 & -\frac{E_x}{c} \\ -B_z & -\frac{E_y}{c} & \frac{E_x}{c} & 0 \end{pmatrix} \quad (23b)$$

Then, dividing tensors (21a-b) by 0 (to zero electromagnetic field in space-time) in $\mathbf{C}^4/0$ we obtain diagonal matrix with 1 elements on the diagonal and all other elements of *numerical value* 0.

$$F_{\mu\nu}/\bar{O} = G^{\mu\nu}/\bar{O} = \eta^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (23c)$$

It is the metric space-time tensor for the Pythagorean interval with all diagonal elements equal to 1 (cf. (17)). Its non-diagonal elements have a total numerical value of 0 and the internal structure $O_B = \{iB_t, -B_t, B_t, -iB_t\}$ or $O_E = \{iE_t, -E_t, E_t, -iE_t\}$, $t \in \{x, y, z\}$. This final structure is responsible for the existence of the zero point energy (vacuum energy)⁵⁵.

As we know, the curvature of two dimensional plane which lies in a three dimensional space embedded in four dimensions R^4 could be described by 4×4 matrix

$$H_{k\tau} = \begin{pmatrix} 0 & k_x & k_y & k_z \\ -k_x & 0 & \tau_z & -\tau_y \\ -k_y & -\tau_z & 0 & \tau_x \\ -k_z & \tau_y & -\tau_x & 0 \end{pmatrix}, \quad (23d)$$

where: k_t ($t = x, y, z$) is the curvature of the plane projected onto the space normal, tangent and 'cotangent' vectors; τ_t measures the rate of change of the space normal around the plane's normal, tangent and 'cotangent' [*scilicet*: perpendicular to the tangent one].

Identifying k_t with B_t and τ_t with $-E_t/c$ we obtain from tensor (21d) the dual electromagnetic field tensor (21b). Now we can interpret the electromagnetic field tensor as a tensor describing the curvature of a cyclic 2D time ('the plane-time')⁵⁶ in 3D space embedded in 4D space-time. Note that the oscillation of any electromagnetic wave is a cyclic process.

Example 6. Internal structure of photon

The relationship between electric charge and mass (20c) implies a diagram of the photon's internal structure. It's presented it in the picture below.

⁵⁵ Sciamia, D. W. "The Physical Significance of the Vacuum State of a Quantum Field", In: Saunders, Simon; Brown, Harvey R. (eds.), *The Philosophy of Vacuum*, Oxford University Press, 1991.

⁵⁶ The existence of a cyclic ('closed') time is implied by the fact, that every measure of time is based on certain cyclic processes. Compare also real and imaginary time axes and 2T-physics [see: R. E. Nusenoff., "Two dimensional time", *Philosophical Studies*, Volume 29, Number 5/May 1976; I. Bars, C. Deliduman, O. Andreev, "Gauged duality, conformal symmetry and space-time with two times", *Phys.Rev. D* 58:066004,1998].

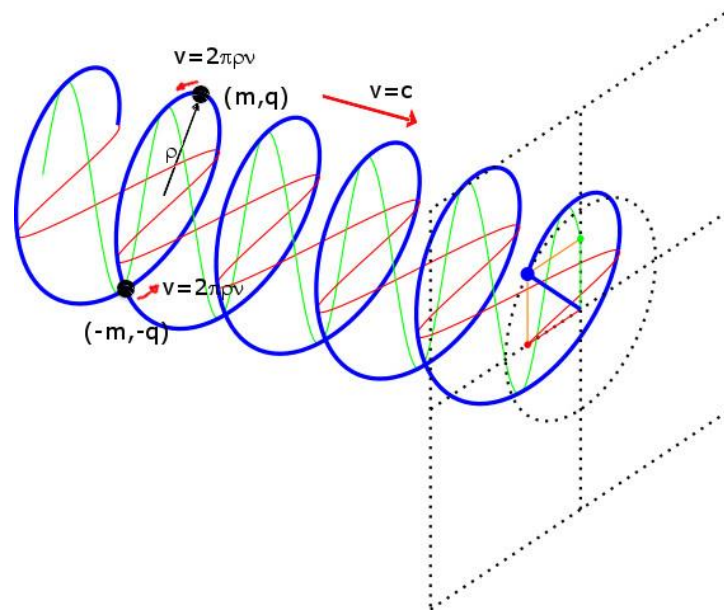


Figure 8 | Model of a classically understood photon (electromagnetic wave) as a result of the motion of a system of two particles – an electron (or positron) and a negpositron (or negelectron) – with masses (respectively) m and $-m$ and electric charges $q \sim im$ and $-q \sim -im$.⁵⁷

The main symmetries

- Electric field - swirl of space reflected in time.
- Magnetic field - swirl of time reflected in space.
- Electromagnetic field - space reflected in motion (i.e. the swirl of space reflected in space-time with a timeline as a projective line).⁵⁸

Basic consequences

- In the figure above: $\rho = l_p = 0$. This implies the zero radius of photon. That is in line with the current state of knowledge in quantum mechanics.
- Also, the total rest mass and the total charge of the photon are - as can be seen - zero.
- The constant linear velocity of a photon ($v_2 = c$) comes from the motion of a system of particles with opposite masses and a total of zero inertia.
- A positive mass particle - as moving at an overall speed greater than the speed of light - is to be considered a tachyon, a negative mass particle - moving at the same speed but in negative time - is to be considered a tardion.
- The frequency of the electromagnetic wave (photon) ν is the same as the frequency of the orbital motion of the abovementioned constituent particles.

⁵⁷ Cf. CALDIROLA, P. "The Introduction of the Chronon in the Electron Theory and a Charged-Lepton Mass Formula", *LETTERE AL NUOVO CIMENTO*, VOL. 27, N. 8 (23 Febbraio 1980).

⁵⁸ Cf. the conception of José Garrigues Baixauli: "Everything is reduced by one space only. Increasing space with velocity originates time. The rotation of space gives rise to the properties of the particles: electric charge, mass, frequency, spin ..." [*What is Space and Time?*, https://www.academia.edu/s/f4ec613ea1#comment_825569].

f) The above model qualitatively explains the Faraday effect.⁵⁹

g) If the whole system moves in space-time in the direction of the r axis from Fig. 49, it is identical with the photon, if along the ct axis - it is a chronon, i.e. a time quantum⁶⁰. The conversion of the sign of the component charges q_1 and q_2 leads to the transformation of the chronon into a photon and vice versa. It can take place in a sufficiently strong alternating magnetic field. In the model of time as the curvature of space, chronon should be equated with the graviton

h) According to point d), both corpuscular components of the photon have in their motion the opposite temporal turn: a particle with a positive mass moves in time from the past into the future, and a particle with a negative mass - from the future into the past. The first of these particles carries information about the past and is responsible for the existence of the cause and effect (causal) aspect of the universe. The second one is a carrier of information about the future and is responsible for the teleological aspect, allowing Nature to pursue its goals and implement them. The light as such is therefore *in extenso* the eternal present - it is a moving border between the future and the past.

Some simple consequences. New source of energy, time machine and antigravity

If every photon is a *chronon* overturned, then every quantum of energy is an inside out quantum of time [what is hidden in time becomes externalized in space]. Then each energy quantum will be a time quantum reversed in space.

Let us take the known formula for the energy of a quantum q of an electromagnetic field with a frequency ν :

$$E_q = h * \nu.$$

Time frequency is a characteristic of any structure that is periodic across position in time. The time frequency is a measure of how often sinusoidal components (as determined by the Fourier transform) of the structure repeat per unit of time. The SI unit of time frequency is cycles per s. In image-processing applications, time frequency is often expressed in units of cycles per s or equivalently line pairs per s.

What is the simplest time periodic [i.e. cyclic] structure in the universe [in 3D space]? This is a quantum time circle in 3D space, i.e. a

$$4\pi t_q^2.$$

sphere. Then the simplest time periodic structure (with)in time (the simplest time periodic structure as a part of time) will be

$$\nu = \frac{t_q}{4\pi t_q^2}.$$

Where ν is an oscillation in 3D physical space. Thus

$$E_q = h\nu = \frac{h}{4\pi t_q}.$$

And from this we have

⁵⁹ Faraday, M. *Faraday's Diary*, Volume IV, Nov. 12, 1839 - June 26, 1847 (Thomas Martin ed.). London: George Bell and Sons, Ltd., 1933. For the mentioned discovery see #7504, 13 Sept. 1845 to #7718, 30 Sept. 1845.

⁶⁰ See: Farias, R.A.H.; Recami, E. (1997-06-27). "Introduction of a Quantum of Time ('chronon'), and its Consequences for Quantum Mechanics", arXiv:quant-ph/9706059.

$$E_q \cdot t_q = \frac{h}{4\pi}. \quad (25a)$$

The (25a) formula is in fact the formula for the transfer of energy into time and *vice versa* on quantum level.

Since the energy of a physical system is always greater than or equal to the energy of a single quantum and the (period of) time of a physical process – greater than a quantum of time, we have the Heisenberg uncertainty principle⁶¹

$$((E \geq E_q) \& (t \geq t_q)) \xrightarrow{(25a)} (E \cdot t \geq \frac{h}{4\pi}).^{62} \quad (25b)$$

where h is a transposition constant (Planck constant) whose magnitude depends on the choice of units.

The change in the strong magnetic field of the direction of the movement of time particles (chronons) from temporal to spatial - i.e. the conversion of *chronons* into photons - will result in the production of energy from time, according to the relation

$$E \cdot t = const.$$

This process may be reversible.

In addition, the breakdown - also in a strong magnetic field - of the photon into a positron and a *negaton* (an electron with a negative mass), or (respectively) into a *nesitron* (i.e. a positron with negative mass) and electron, will allow to obtain negmatter and - as its consequence - antigravity.⁶³

The existence of a negative mass allows (technically) unlimited gaining of energy from mass, according to the relationship

$$Mc^2 = \tilde{M}c^2 + E,$$

where $E > Mc^2$, $\tilde{M} < 0$. The negative mass \tilde{M} obtained as a result of such a process can be used to obtain the anti-gravity effect and to erect ultra-light building structures.

The above predicted model can explain both gamma and X-ray emission by magnetars having a strong magnetic field [$B > 10^{10} T$] magnetars^{64, 65, 66}, fluorescence effect in magnetic fields (MFE)^{67, 68}

⁶¹ Heisenberg, W. "Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik", *Zeitschrift für Physik*, 43 (1927): 172–198.

⁶² As Stephen Hawking wrote: „Yet it seems that the uncertainty principle is a fundamental feature of the universe we live in. A successful unified theory must therefore necessarily incorporate this principle” [*Theory of Everything*, op. cit.].

⁶³ As I finish the first volume of this book, the communicate of the discovery of an electron with a negative mass appeared in *Nature*, which of course confirms the presented concept: Lin, K.- Q. et al., "Narrow-band high-lying excitons with negative-mass electrons in monolayer WSe₂", *Nature Communications*, Published: 17 September 2021.

⁶⁴ Tiengo, A., Schartel, N. "Weakling magnetar reveals hidden strength", ESA Science&Technology, 14 August 2013, <http://sci.esa.int/xmm-newton/52772-weakling-magnetar-reveals-hidden-strength/>.

⁶⁵ Lyubarsky, Y. *A model for fast extragalactic radio bursts*, arXiv:1401.6674v1 [astro-ph.HE].

⁶⁶ Greiner, J., Mazzali, P.A., "A very luminous magnetar-powered supernova associated with an ultra-long γ-ray burst", *Nature* volume 523, pages 189–192 (09 July 2015).

⁶⁷ Kuznetsov, A.I. et. al., "Magnetic light", *Scientific Reports*, **volume 2**, Article number: 492 (2012), <https://www.nature.com/articles/srep00492>.

⁶⁸ Ohta, N. *Magnetic Field Effects on Fluorescence in Isolated Molecules with the Intermediate Level Structure of Singlet–Triplet Mixed States*, <https://pubs.acs.org/doi/10.1021/jp9517514>.

and phenomena such as polar aurora⁶⁹. Until recently, the latter phenomenon was considered to be well explained, but the currently adopted models of it imply the dependence of the process of lighting the aurora on the intensity of the solar wind⁷⁰. However, the intensity of this solar wind near Jupiter is almost 30 times lower than those near Earth⁷¹ but Jupiter has auroras 10⁴ times more intense than those on the Earth – which corresponds to about 20,000 times higher intensity of the magnetic field of Jupiter in relation to the magnetic field of the Earth – and does not seem to be dependent on the concentration of solar wind particles⁷². Similar auroras - the magnitude of which appear to depend only on the values of the respective magnetic fields - are observed on Saturn^{73, 74}, Uranus^{75, 76}, Neptune^{77, 78} and some of extrasolar planets^{79, 80}.

The principle of equivalence of time and energy also explains the phenomenon of sonoluminescence⁸¹ as the conversion of the period of vibration of a sound wave into the energy of a light wave.

Conclusions. The Author also touches on many other topics in his book. In particular, He presents new discoveries and observations about the history of science from the point of view of people with high cognitive abilities (high IQ). He reconstructed, among other things, the Pythagorean-Platonic method of dialectical thinking, the lost cosmology of Philolaus (mentioned in his epoch-making work by Nicolaus Copernicus) and reassembles and develops the Pythagorean arithmetic of language, which - according to the Author - was probably the greatest intellectual achievement of mankind so far. The Author also presents a synthetic cosmological model and the basics of the theory of everything. A second volume, announced at the end of the book, contains new, detailed results on obtaining energy from time (conversion of chronons into photons) or interstellar travel (chronophotonic engine).

Such science is the most effective both cognitively (enabling a deep understanding of reality) and due to its technical applications (unprecedented technological development).

⁶⁹ “NASA snaps unprecedented image of auroras on gas giant Uranus”, Last updated on April 13th, 2017 at 12:48 pm by Tibi Puiu, <https://www.zmescience.com/space/aurora-on-neptune/>.

⁷⁰ *Northern Lights: What Causes the Aurora Borealis & Where to See It*, By SPACE.com Staff October 11, 2017 Science & Astronomy, <https://www.space.com/15139-northern-lights-auroras-earth-facts-sdcmp.html>.

⁷¹ https://www.researchgate.net/figure/Jovian-Saturn-Uranus-and-Neptune-agnetospheres_fig7_258925679.

⁷² <https://www.nasa.gov/feature/goddard/2016/hubble-captures-vivid-auroras-in-jupiter-s-atmosphere>.

⁷³ Kettley, S. *Northern lights on Saturn in pictures: Hubble snaps breathtaking aurora on ringed planet*, <https://www.express.co.uk/news/science/1014021/Northern-lights-Saturn-NASA-pictures-Hubble-space-telescope>.

⁷⁴ <https://asterisk.apod.com/viewtopic.php?t=21279>.

⁷⁵ *Rare Photo: Auroras on Uranus Spotted by Hubble Telescope*, By SPACE.com Staff April 13, 2012 Science & Astronomy, <https://www.space.com/15270-auroras-uranus-hubble-telescope-photos.html>.

⁷⁶ King, B. *Hubble Sees Intense Auroras on Uranus*, April 11, 2017, <https://www.universetoday.com/134965/hubble-sees-intense-auroras-uranus/>.

⁷⁷ Cheng, A. F. *Global magnetic anomaly and aurora of Neptune*, <https://ntrs.nasa.gov/search.jsp?R=19900065487>.

⁷⁸ Cheng, A. photonF. “Triton torus and Neptune aurora”, *Geophys. Res. Lett.*, 1990, 17:1669 – 1672.

⁷⁹ Kao, M.M. et. al., “The Strongest Magnetic Fields on the Coolest Brown Dwarfs”, *The Astrophysical Journal Supplement Series, Volume 237, Number 2*.

⁸⁰ Kohler, S. “Can We Detect Auroral Emission from Proxima b?”, 14 November 2017, AAS Nova, <https://aasnova.org/2017/11/14/can-we-detect-auroral-emission-from-proxima-b/>.

⁸¹ Jarman, P. “Sonoluminescence: a discussion”, *The Journal of the Acoustical Society of America*, 32(11):1459–1462, 1960